

## Minimizing the number of states – implication tables

Example state table:

Row reduction: two states are equivalent if they the same N.S. and the same output for any input



| PS | NS  |     | Z   |     |
|----|-----|-----|-----|-----|
|    | X=0 | X=1 | X=0 | X=1 |
| A  | B   | C   | 0   | 1   |
| B  | A   | E   | 1   | 0   |
| C  | D   | A   | 0   | 1   |
| D  | C   | E   | 1   | 0   |
| E  | A   | F   | 1   | 0   |
| F  | E   | F   | 0   | 1   |

To minimize the number of states, we will identify “equivalent states” and eliminate any redundancy found. Two states are equivalent if they have equivalent next states and the same output for each possible input condition. To find equivalent states we will create an “implication table” which looks at pairs of states and identifies which states have to be equivalent if this pair is to be equivalent. We can use a table to hold information about each pair.

|   | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A |   |   |   |   |   |   |
| B |   |   |   |   |   |   |
| C |   |   |   |   |   |   |
| D |   |   |   |   |   |   |
| E |   |   |   |   |   |   |
| F |   |   |   |   |   |   |

To investigate all possible pairs, we could use a square table such as this to record information about pairs of states. But note every pair represented in the upper right “triangle” of the table is also listed in the lower left “triangle” of the table. Furthermore, the diagonal of the table will only present information about a state being equivalent to itself. Only the part of the table in **bold** is needed to investigate all possible pairs of states:

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| B |   |   |   |   |   |
| C |   |   |   |   |   |
| D |   |   |   |   |   |
| E |   |   |   |   |   |
| F |   |   |   |   |   |
|   | A | B | C | D | E |

So this is all we need to hold information about all possible pairs of states. There is one cell for each pair of states.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| B | X |   |   |   |   |
| C |   | X |   |   |   |
| D | X |   | X |   |   |
| E | X |   | X |   |   |
| F |   | X |   | X | X |
|   | A | B | C | D | E |

For each pair that has any difference in the output, we’ll place an X in the table to indicate that pair cannot be equivalent. We’ll do this first.

In the following we use  $A \equiv F$  as a shorthand to say  $A$  is equivalent to  $F$

Remaining pairs have the same output, so they could possibly be equivalent, but that is only if their “next states” are equivalent. For example states  $A$  and  $F$  would be equivalent ( $A \equiv F$ ) if it turns out that  $B \equiv E$  (for  $X=0$ ) and  $C \equiv F$  (for  $X=1$ ). In other words, we can say that state  $A$  being equivalent to state  $F$  implies  $B \equiv E$  and  $C \equiv F$ . If it turns out that either  $B$  is not equivalent to  $E$  or  $C$  is not equivalent to  $F$ , then it must be that  $A$  is not equivalent to  $F$ . We can enter these two “implied pairs” into the implication table at the  $A$ - $F$  intersection.

|   |                              |                                    |                              |  |   |
|---|------------------------------|------------------------------------|------------------------------|--|---|
| B | X                            |                                    |                              |  |   |
| C | $B \equiv D$                 | X                                  |                              |  |   |
| D | X                            | $A \equiv C$                       | X                            |  |   |
| E | X                            | <del><math>E \equiv F</math></del> | X                            | <del><math>A \equiv C</math><br/><math>E \equiv F</math></del> |   |
| F | $B \equiv E$<br>$C \equiv F$ | X                                  | $D \equiv E$<br>$A \equiv F$ | X  | X |
|   | A                            | B                                  | C                            | D  | E |

Other implied pairs are filled in similarly, but note that some implied pairs are trivial and need not be listed. For example  $A \equiv C$  has the implied pairs of  $B \equiv D$  and  $A \equiv C$ . Of course if  $A$  is equivalent to  $C$  then  $A$  is equivalent to  $C$ , so the  $A \equiv C$  implied pair does not need to be listed at the  $A$ - $C$  intersection. As another example,  $B \equiv D$  has the implied pairs  $A \equiv C$  and  $E \equiv F$ , but this second pair is always true and need not be listed. The full implication table is therefore as given here.

Now we can eliminate pairs that cannot be equivalent because one or more of their implied pairs are not equivalent. Here:

(looking at the  $B$ - $E$  intersection)  $B$  cannot be equivalent to  $E$  as  $E$  is not equivalent to  $F$

(looking at the  $D$ - $E$  intersection)  $D$  cannot be equivalent to  $E$  as  $E$  is not equivalent to  $F$

These two cells,  $B$ - $E$  and  $D$ - $E$ , are therefore  $X$ -ed out as they are not equivalent pairs of states

|   |  |              |  |   |   |
|---|--|--------------|--|---|---|
| B | X  |              |  |   |   |
| C | $B \equiv D$   | X            |  |   |   |
| D | X  | $A \equiv C$ | X  |   |   |
| E | X  | X            | X  | X |   |
| F | <del><math>B \equiv E</math><br/><math>C \equiv F</math></del> | X            | <del><math>D \equiv E</math><br/><math>A \equiv F</math></del> | X | X |
|   | A  | B            | C  | D | E |

Now from this updated table,

(looking at the A-F intersection) A cannot be equivalent to F as B is not equivalent to E  
 (looking at the C-F intersection) C cannot be equivalent to F as D is not equivalent to E  
 Cells, A-F and C-F, are therefore X-ed out.

|   |       |       |   |   |   |
|---|-------|-------|---|---|---|
| B | X     |       |   |   |   |
| C | B ≡ D | X     |   |   |   |
| D | X     | A ≡ C | X |   |   |
| E | X     | X     | X | X |   |
| F | X     | X     | X | X | X |
|   | A     | B     | C | D | E |

→ A ≡ C  
 B ≡ D

At this point we've crossed out everything that is not equivalent. What we have left says that A ≡ C as long as B ≡ D, and B ≡ D as long as A ≡ C. This works, so

A ≡ C  
 B ≡ D

and we can redraw the state table as follows:

| PS | NS  |     | Z   |     |
|----|-----|-----|-----|-----|
|    | X=0 | X=1 | X=0 | X=1 |
| A  | B   | A   | 0   | 1   |
| B  | A   | E   | 1   | 0   |
| E  | A   | F   | 1   | 0   |
| F  | E   | F   | 0   | 1   |