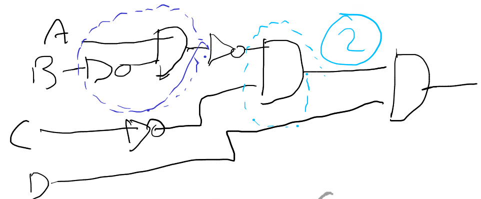
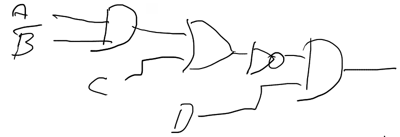
Demorgan's

$\overline{X+Y} = \bar{X}\cdot\bar{Y}$





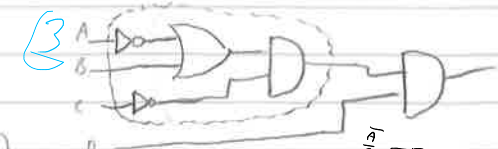
4 gates + 7 input# = 11

Boolean Algebra for Synthesis

get rid of "big bars"

① $f = \overline{A\bar{B} + C} \cdot D$

② $\overline{A\bar{B}} \cdot \bar{C} \cdot D$

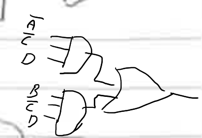


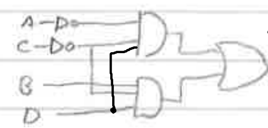
③ $(\bar{A}+B)\cdot\bar{C}\cdot D$

Distributive law

$\boxed{f = \bar{A}\bar{C}D + B\bar{C}D}$

SOP
Sum of Products





* Also use Boolean Algebra $= \bar{A}\bar{C}D(B+\bar{B}) + B\bar{C}D(A+\bar{A})$

$\bar{A}B\bar{C}D + \bar{A}\bar{B}\bar{C}D + AB\bar{C}D + \cancel{\bar{A}B\bar{C}D}$

same

do this for SOP also

canonical minterm form $f = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D$

0001 0101 1101

$f = m_1 + m_5 + m_{13}$

A B C D	f	A+B+C+D	A+B+$\bar{C}$+D
0 0 0 0	0	0	1
0 0 0 1	1	1	1
0 0 1 0	0	1	0
0 0 1 1	0	1	1
0 1 0 0	0	1	1
0 1 0 1	1	1	1
0 1 1 0	0	1	1
0 1 1 1	0	1	1
1 0 0 0	0	1	1
1 0 0 1	0	1	1
1 0 1 0	0	1	1
1 0 1 1	0	1	1
1 1 0 0	0	1	1
1 1 0 1	1	1	1
1 1 1 0	0	1	1
1 1 1 1	0	1	1

one measure of cost = # gates + # inputs to all gates (include inverters)

$A\bar{B}C$
101 $= m_5$

Original circuit cost = ~~13~~ 11 ← multilevel

→ 11 if $\bar{B}$ is free

SoP cost = 14 ← 2 level

canonical cost = 25

maxterm form $f = \prod M(0,2,3,4,6,7,8,9,10,11,12,14,15)$

(A+B+C+D)
0 0 0 0

$f = (A+B+C+D)(A+B+\bar{C}+D)(A+B+\bar{C}+\bar{D})(A+\bar{B}+C+D)(A+\bar{B}+\bar{C}+D)$

0010

$(A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+D)$

$(x+y)(x+\bar{y}) = x$ $\quad (\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$

A+B+D

* $f = (\bar{A}+B)\,\bar{C}\,D$ ← this is POS.

easy to get this from SOP using algebraic manipulation

	A			
	0	0	0	0
	1	1	1	0
C	0	0	0	0
C	0	0	0	0

B, D

Note $\bar{f}$ = complement of f (in truth table 1 ⇄ 0)

↑ minterm rows of $\bar{f}$ = maxterm rows of f

treat zeros of f as ones (this is $\bar{f}$)

Demorgan's

$\bar{f} = A\bar{B} + C + \bar{D}$

$\bar{\bar{f}} = \overline{(\bar{A}+B)\cdot\bar{C}\cdot D}$

$f = (\bar{A}+B)\cdot\bar{C}\cdot D$

Also maxterms

$\bar{f} = \Sigma m(0,2,3,4,6,7,8,9,10,11,12,14,15)$

$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \ldots$

$f = (A+B+C+D)(A+B+\bar{C}+D)(\ldots$

$\bar{f} = \bar{D} + C + A\bar{B}$

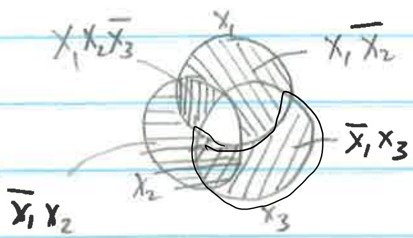
$f = (\bar{A}+B)\,\bar{C}\,D$

5a alt

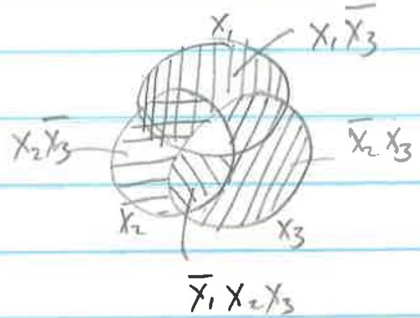
multiple ways to solve

2.7a in Text

$$\bar{x}_1 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 + x_1 \bar{x}_2 = \bar{x}_2 x_3 + x_1 \bar{x}_3 + x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3$$



	$x_1\ x_2\ x_3$	f
m_0	0 0 0	0
m_1	0 0 1	1
m_2	0 1 0	1
m_3	0 1 1	1
m_4	1 0 0	1
m_5	1 0 1	1
m_6	1 1 0	1
m_7	1 1 1	0



	$x_1\ x_2\ x_3$	f
m_0	0 0 0	0
m_1	0 0 1	1
m_2	0 1 0	1
m_3	0 1 1	1
m_4	1 0 0	1
m_5	1 0 1	1
m_6	1 1 0	1
m_7	1 1 1	0

$$\bar{x}_1 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 + x_1 \bar{x}_2$$

$$\bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 \bar{x}_3 + \cancel{\bar{x}_1 x_2 x_3} + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3$$

001   011   110

m_1 m_3 m_6 m_2 m_3 m_4 m_5

$$\bar{x}_2 x_3 + x_1 \bar{x}_3 + x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3$$

$$\bar{x}_1 \bar{x}_2 x_3 + x_1 \bar{x}_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 \bar{x}_3 + \cancel{x_1 x_2 \bar{x}_3} + \bar{x}_1 x_2 x_3$$

m_1 m_5 m_4 m_6 m_2 m_6 m_3

same

Canonical minterm form

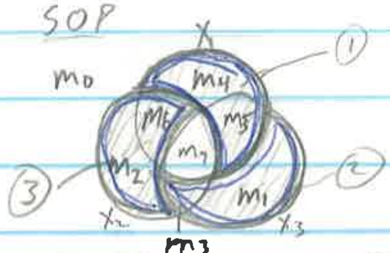
$$f = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$$

$$= m_1 + m_2 + m_3 + m_4 + m_5 + m_6$$

$$= \Sigma m(1,2,3,4,5,6)$$

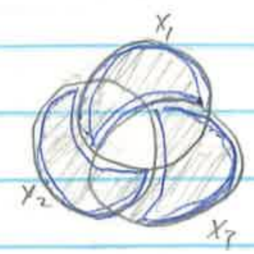
Two functions are equivalent iff their canonical forms are the same

minimal SOP



Combining theorem
product terms that differ in only one variable can be combined

ab*c + abc = ac



$$f = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$$

$\bar{x}_1 x_3$ $x_1 \bar{x}_2$ $x_2 \bar{x}_3$

$$f = \bar{x}_1 x_3 + x_1 \bar{x}_2 + x_2 \bar{x}_3$$

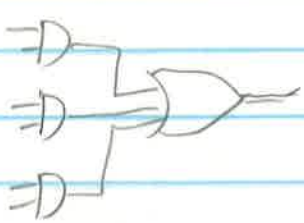
Alternately

$$\bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$$

$\bar{x}_1 x_2$ $x_1 \bar{x}_3$ $\bar{x}_2 x_3$

$$f = \bar{x}_1 x_2 + \bar{x}_2 x_3 + x_1 \bar{x}_3$$

cost



cost = 4 + 9 = 13

$x + x = x$

gates + # inputs
4   9

another measure = # gates

cost of canonical = 7 + 24 = 31

cost of original = 5 + 13 = 18

don't count inverters on inputs

$\Sigma m(1,2,3,4,5,6)$

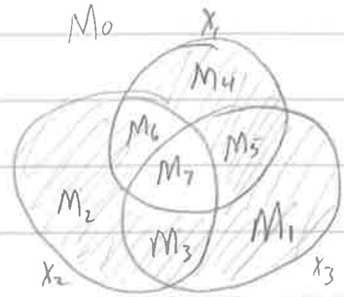
Canonical Maxterm form

$f = \underline{\Pi}\ \underline{M}(0,7)$

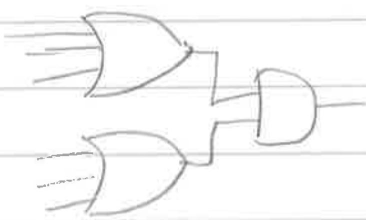
$\underline{M_0}$ $\underline{M_7}$

$(x_1 + x_2 + x_3)\ (\overline{x}_1 + \overline{x}_2 + \overline{x}_3)$

canonical maxterm form
<u>and</u> minimal POS



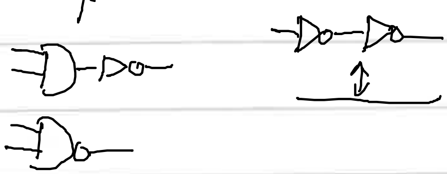
M_0 + M_7 are not adjacent so they cannot be combined



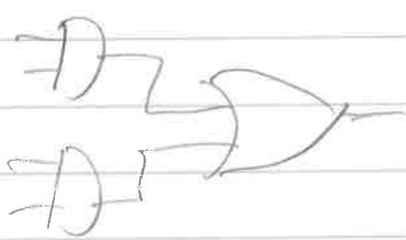
cost = 3 + 8 = 11 Cheaper!

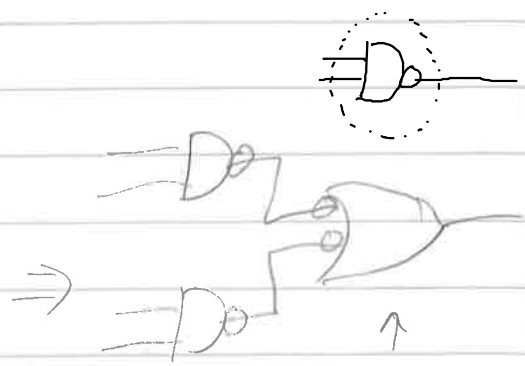
lowest cost circuit

NAND - NAND



SOP

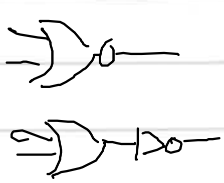


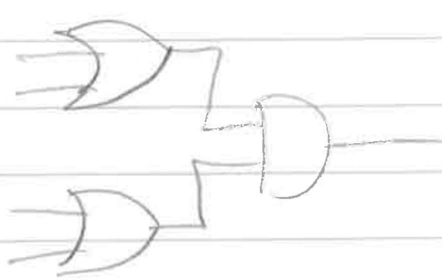


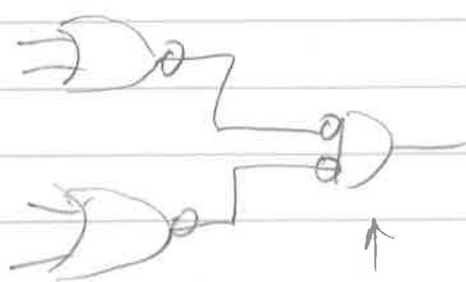
NAND

POS

NOR - NOR







NOR

Turn SOP into POS (easier to turn POS into SOP so do dual)

DUAL $(\overline{X_1} + X_3)(X_1 + X_2 + \overline{X_3})(\overline{X_1} + X_2)(X_1 + \overline{X_2})$ dual of original

$(\overline{X_1}X_2 + \overline{X_1}\,\overline{X_3} + X_1X_3 + X_2X_3) \cdot (\overline{X_1}\,\overline{X_2} + X_1X_2)$

$\overline{X_1}\,\overline{X_2}\,\overline{X_3} + X_1X_2X_3$

dual back

$(\overline{X_1} + \overline{X_2} + \overline{X_3})(X_1 + X_2 + X_3)$

without Taking Dual

SOP $\overline{X_1}X_3 + X_1X_2\overline{X_3} + \overline{X_1}X_2 + X_1\overline{X_2}$

$(\overline{X_1} + X_2)(\overline{X_1} + \overline{X_3})(X_1 + X_3)(X_2 + X_3) + (\overline{X_1} + \overline{X_2})(X_1 + X_2)$

POS $(\overline{X_1} + \overline{X_2} + \overline{X_3})(X_1 + X_2 + X_3)$