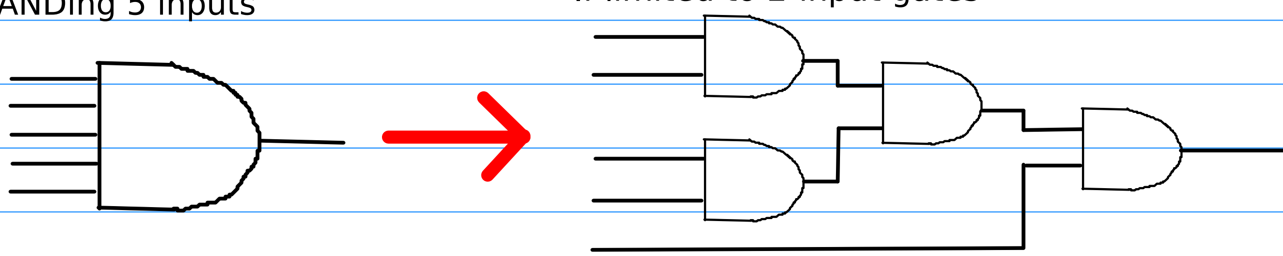
Multi-level synthesis

So far we've looked at mostly SOP and POS (two-level circuits)
Can (but not always) have less delay
(output changes in response to an input change)

BUT:
Two level circuits often have higher "fan-in" (number of inputs to a gate)
-- higher fan-in can mean more delay in the output
-- higher fan-in may not be possible, or may be more costly in some cases
e.g., an FPGA may use a Look Up Table, limiting to two inputs per gate

E.g., ANDing 5 inputs

If limited to 2-input gates



So you may end up with multiple levels anyway.

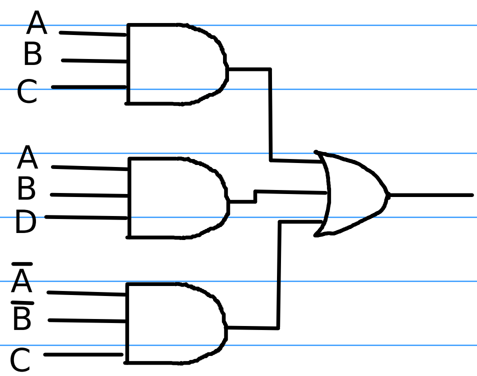
Increasing the number of levels beyond two can often lower the "cost"
and may automatically decrease the maximum fan-in required:

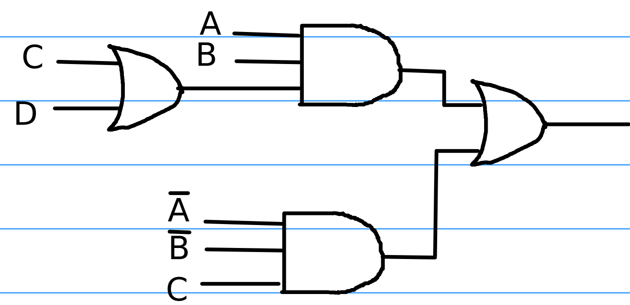
Two approaches: factoring and functional decomposition

Factoring:

$ABC + ABD + \bar{A}\bar{B}C$

$AB(C + D) + \bar{A}\bar{B}C$





Cost = 4 + 12 = 16

Cost = 4 + 10 = 14

With multiple outputs you can share terms:

SOP

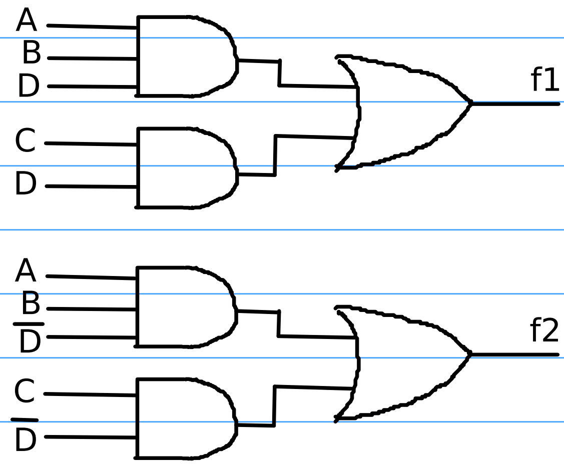
$f1 = ABD + CD$

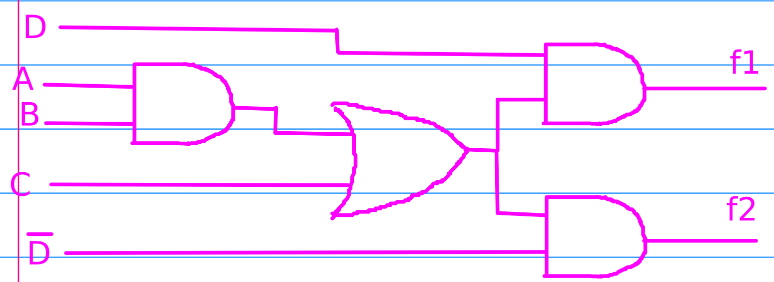
$= (AB + C)\,D$

Note the common term when factored

$f2 = A\overline{B}D + \overline{C}D$

$= (AB + \overline{C})\,D$





Cost = 6 + 14 = 20

Cost = 4 + 8 = 12

Maximum fan-in is 2

Functional Decomposition:

Maximum fan-in is 4

Example in the text

SOP cost = 5 + 16 = 21

$f = \overline{x1}\,x2\,x3 + x1\,\overline{x2}\,x3 + \overline{x1}\,\overline{x2}\,x4 + x1\,x2\,x4$

EOR

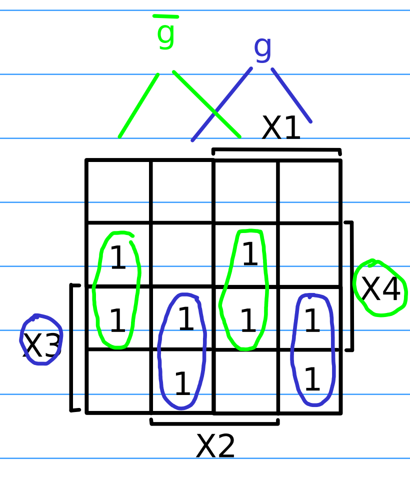
$g = \overline{x1}\,x2 + x1\,\overline{x2} = x1 \oplus x2$

Cost = 3 + 6 = 9

$f = g\,x3 + \overline{g}\,x4$

Cost = 3 + 6 = 9

$\overline{g}$ cost = 1 + 1 = 2

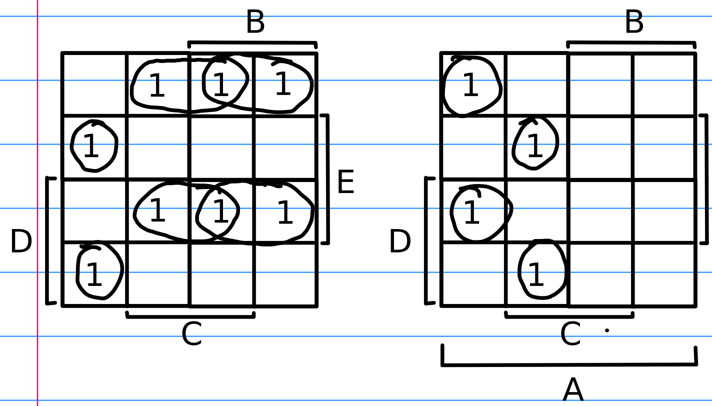


Total cost = 9 + 9 + 2 = 20

Note that the maximum fan-in is 2

Note that these problems may look "contrived", but real-world problems often have "patterns" rather than being "random" due to where they come from. Real examples often have many more variables and many, many things to try. Even CAD tools cannot try them all, so they have to resort to "heuristics".

Example 5-variable problem



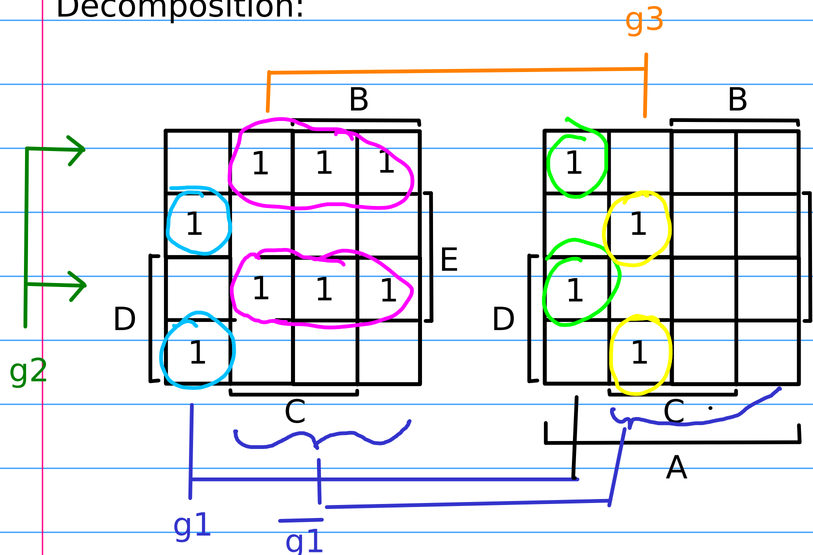
Minimal SOP:

$$f = \bar{A}B\bar{D}\bar{E} + \bar{A}C\bar{D}\bar{E} + \bar{A}\bar{B}\bar{C}\bar{D}E$$
$$+ \bar{A}BDE + \bar{A}CDE + \bar{A}\bar{B}\bar{C}D\bar{E}$$
$$+ A\bar{B}\bar{C}\bar{D}\bar{E} + A\bar{B}\bar{C}DE$$
$$+ A\bar{B}C\bar{D}E + A\bar{B}CD\bar{E}$$

Cost: 4 4-input ANDs = 4 + 4*4 = 20
6 5-input ANDs = 6 + 6*5 = 36
1 10-input OR = 1 + 10 = 11
Total cost = 67

Maximum fan-in is 10

Decomposition:



$\overline{g1} = B + C$

$g1 = \bar{B}\bar{C}$

Cost = 1 + 2 = 3

$\overline{g1}$ cost = 1 + 1 = 2

$g2 = DE + \bar{D}\bar{E}$

Cost = 3 + 6 = 9

$\overline{g2}$ cost = 1 + 1 = 2

$g3 = \bar{B}C$

Cost = 1 + 2 = 3

$$f = \bar{A}\,\overline{g1}\,g2 + \bar{A}\,g1\,\overline{g2} + A\,g1\,g2 + A\,g3\,\overline{g2}$$

Cost = 5 + 16 = 21

Total cost = 3 + 2 + 9 + 2 + 3 + 21 = 40

Maximum fan-in is 4

How it looks with factoring

$$f = \bar{A}\,\underbrace{(B + C)}_{\overline{g1}}\,\underbrace{(DE + \bar{D}\bar{E})}_{g2} + \bar{A}\,\underbrace{(\bar{B}\bar{C})}_{g1}\,\underbrace{(D\bar{E} + \bar{D}E)}_{\overline{g2}} + A\,\underbrace{(\bar{B}\bar{C})}_{g1}\,\underbrace{(DE + \bar{D}\bar{E})}_{g2} + A\,\underbrace{(\bar{B}C)}_{g3}\,\underbrace{(D\bar{E} + \bar{D}E)}_{\overline{g2}}$$

How about if we instead factored out A / $\bar{A}$?

$$f = \bar{A}(\overline{g1}\,g2 + g1\,\overline{g2}) + A(g1\,g2 + g3\,\overline{g2})$$

Cost = 9 + 18 = 27

Total cost = 3 + 2 + 9 + 2 + 3 + 27 = 46

Maximum fan-in is 2