$\Rightarrow$ (Composite Homogeneous) D-H transformation matrix ${}^{i-1}A_i$ (from $i-1$ to i)

$$
{}^{i-1}A_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Given link parameters $d_i, \theta_i, a_i, \alpha_i$

$$
= \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Remember: For revolute (rotating) joints $\alpha_i\ a_i\ d_i$ are joint parameters (fixed constants) θ_i = joint variable
For prismatic (sliding) joints $\alpha_i\ a_i\ \theta_i$ are joint parameters d_i = joint variable.

Note the inverse $\left[{}^{i-1}A_i\right]^{-1} = {}^{i}A_{i-1} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 & -a_i \\ -\cos\alpha_i \sin\theta_i & \cos\alpha_i \cos\theta_i & \sin\alpha_i & -d_i \sin\alpha_i \\ \sin\alpha_i \sin\theta_i & -\sin\alpha_i \cos\theta_i & \cos\alpha_i & -d_i \cos\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Now can determine ${}^{0}T_i$ ← transformation from base to end effector

$$
{}^{0}T_i = {}^{0}A_1\, {}^{1}A_2\, {}^{2}A_3 \cdots {}^{i-1}A_i = \prod_{j=1}^{i} {}^{j-1}A_j
$$

if "i" ($=n$) is for the end effector (last link) ${}^{0}T_n$ is called the "arm matrix"

Note ${}^{0}T_n$ tells how points in hand (end effector) system transform to base system $\hat{P}_0 = {}^{0}T_6\, \hat{P}_6$

$$
\bar{T} = \begin{bmatrix} {}^{0}\bar{R}_6 & {}^{0}\bar{t}_6 \\ \bar{0} & 1 \end{bmatrix} = \begin{bmatrix} \bar{n} & \bar{s} & \bar{a} & \bar{t} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

n = vector in direction of "palm"
s = vector in direction of finger motion
a = vector in direction of fingers
p = position of hand origin w.r.t. base

Now, if the robot base is related to a reference frame by $\bar{B} = {}^{ref}\bar{A}_0$ and a tool frame is related to the end effector frame by $\bar{H} = {}^{6}\bar{A}_{tool}$

then ${}^{ref}\bar{T}_{tool} = \bar{B}\,{}^{0}T_6\,\bar{H}$

Note.
Computing ${}^{0}T_6 = \prod_{j=1}^{i} {}^{j-1}\bar{A}_j$

1. Compute ${}^{j-1}\bar{A}_j$ and multiply. ← many zero. in each.
2. Hand multiply ${}^{0}\bar{A}_1\,{}^{1}\bar{A}_2\,{}^{2}\bar{A}_3 \Rightarrow {}^{0}\bar{A}_3$ and ${}^{3}\bar{A}_4\,{}^{4}\bar{A}_5\,{}^{5}\bar{A}_6 \Rightarrow {}^{3}\bar{A}_6$

   To get two matrices to be multiplied. $T = {}^{0}\bar{A}_3 \cdot {}^{3}\bar{A}_6$

eg. for PumA:

$$
{}^{0}A_3 = \begin{bmatrix} C_1C_{23} & -S_1 & C_1S_{23} & a_2C_1C_2 + a_3C_1C_{23} - d_2S_1 \\ S_1S_{23} & C_1 & S_1S_{23} & a_2S_1C_2 + a_3S_1C_{23} + d_2C_1 \\ -S_{23} & 0 & C_{23} & -a_2S_2 - a_3S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad C_{ij} = \cos(\theta_i + \theta_j)
$$

$$
{}^{3}A_6 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4S_6 & C_4S_5 & d_6C_4S_5 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & d_6S_4S_5 \\ -S_5C_6 & S_5S_6 & C_5 & d_6C_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Or can multiply all by hand to get a single matrix ${}^{0}A_6 = {}^{0}A_3\,{}^{3}A_6$

Note orientation of hand is described by the R_{33} portion of T
Could convert to Euler angles or axis/angle representations

Position of the hand (origin) is given by the last column — Cartesian coordinates $(P_x, P_y, P_z)^T$

could convert to cylindrical $(rC\alpha, rS\alpha, d)^T$ $\left\{ d = P_z \quad \alpha = \mathrm{atan2}(P_y, P_x) \quad r = \sqrt{P_x^2 + P_y^2} \right\}$

or spherical $(rC\alpha S\beta, rS\alpha S\beta, rC\beta)^T$ $\left\{ r = \sqrt{P_x^2 + P_y^2 + P_z^2} \quad \beta = \cos^{-1}(P_z/r) \quad \alpha = \mathrm{atan2}(P_y, P_x) \right\}$