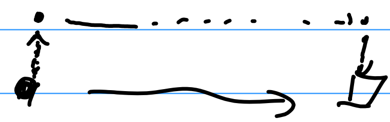
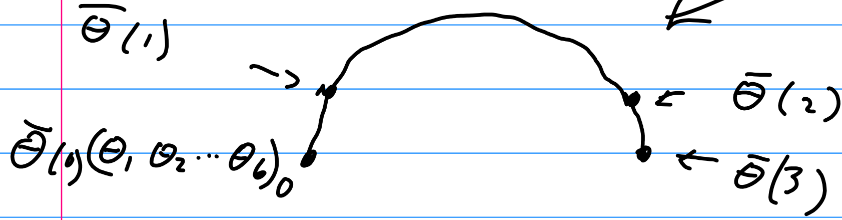
Trajectory planning

① straight lines ← inverse kinematics

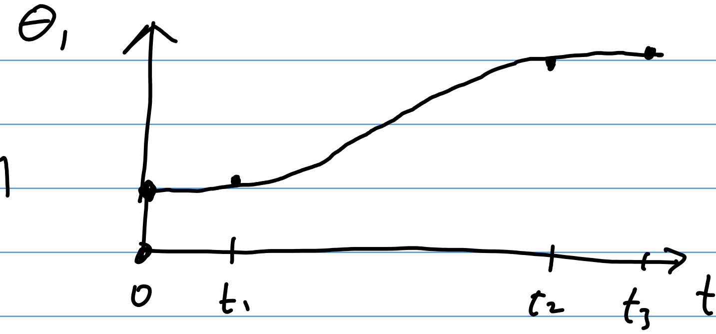
② curve ← Planning in cartesian space

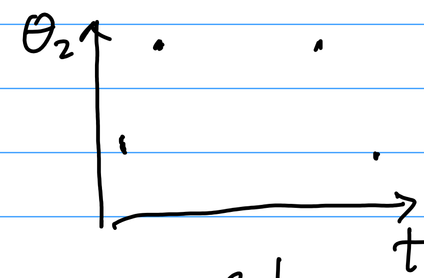




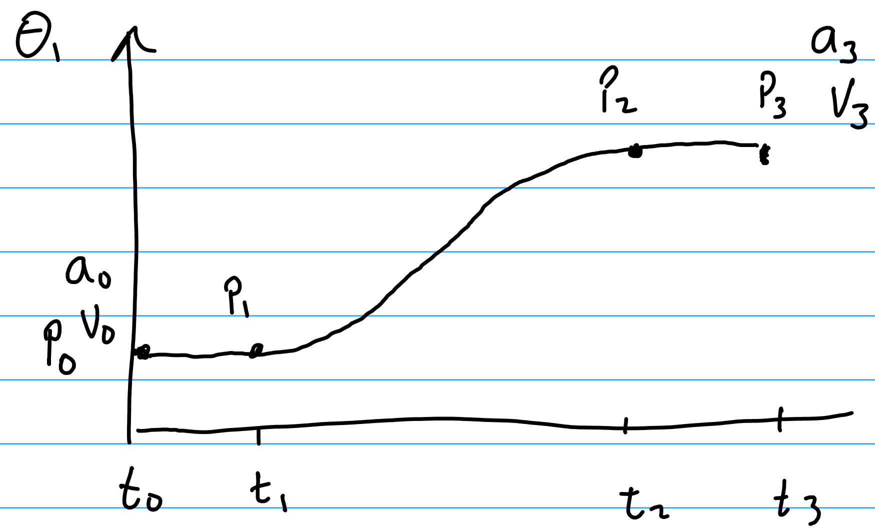
Planning in joint space

Each axis individually





etc



F = ma

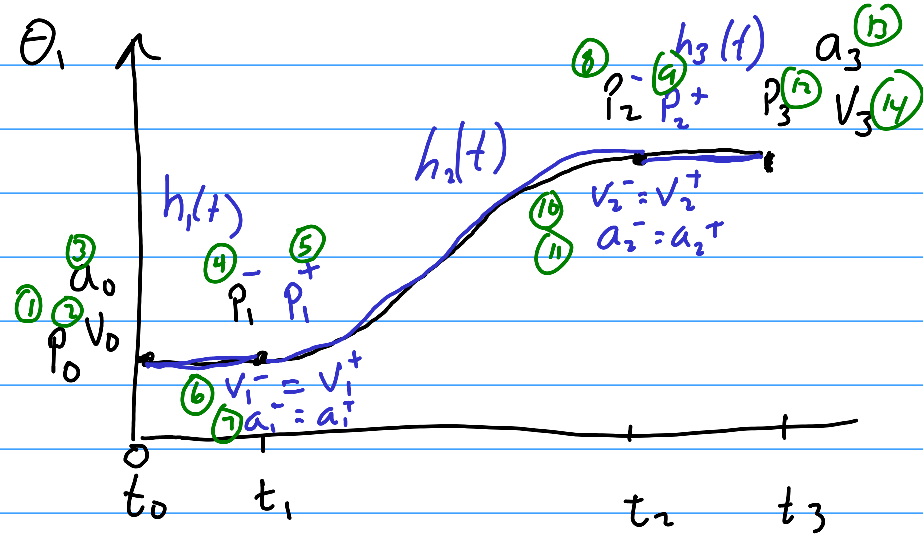
8 - constraints

7^{th} order polynomial has 8 coefficients

↳ Tends to have exteraneous motion
Difficult to find extrema

add constiants

Instead of a single 7^{th} we'll try a spline of 3 polynomials



14 – constraints $\Rightarrow$ 14 coefficients

possibilities

	5 - 1 - 5	X		
common	4 - 3 - 4	trajectory	5+4+5	coefficients
	3 - 5 - 3	"	4+6+4	"
	2 - 7 - 2	X		

we'll 4-3-4

$$h_1(t) = a_{14}t^4 + a_{13}t^3 + a_{12}t^2 + a_{11}t + a_{10}$$
$$\dot{h}_1(t) = 4a_{14}t^3 + 3a_{13}t^2 + 2a_{12}t + a_{11}$$
$$\ddot{h}_1(t) = 12a_{14}t^2 + 6a_{13}t + 2a_{12}$$
$$h_2(t) = a_{23}t^3 + a_{22}t^2 + a_{21}t + a_{20}$$
$$\dot{h}_2(t) = 3a_{23}t^2 + 2a_{22}t + a_{21}$$
$$\ddot{h}_2(t) = 6a_{23}t + 2a_{22}$$
$$h_3(t) = a_{34}t^4 + a_{33}t^3 + a_{32}t^2 + a_{31}t + a_{30}$$
$$\dot{h}_3(t) = 4a_{34}t^3 + 3a_{33}t^2 + 2a_{32}t + a_{31}$$
$$\ddot{h}_3(t) = 12a_{34}t^2 + 6a_{33}t + 2a_{32}$$

P_0 | $h_1(0) = P_0 \Rightarrow a_{10} = P_0$

V_0 | $\dot{h}_1(0) = V_0 \Rightarrow a_{11} = V_0$

a_0 | $\ddot{h}_1(0) = a_0 \Rightarrow 2a_{12} = a_0 \Rightarrow a_{12} = a_0/2$

P_1^- | $h_1(t_1) = P_1 \Rightarrow a_{14}t_1^4 + a_{13}t_1^3 + a_{12}t_1^2 + a_{11}t_1 + a_{10} = P_1$

P_1^+ | $h_2(t_1) = P_1 = a_{23}t_1^3 + a_{22}t_1^2 + a_{21}t_1 + a_{20} = P_1$

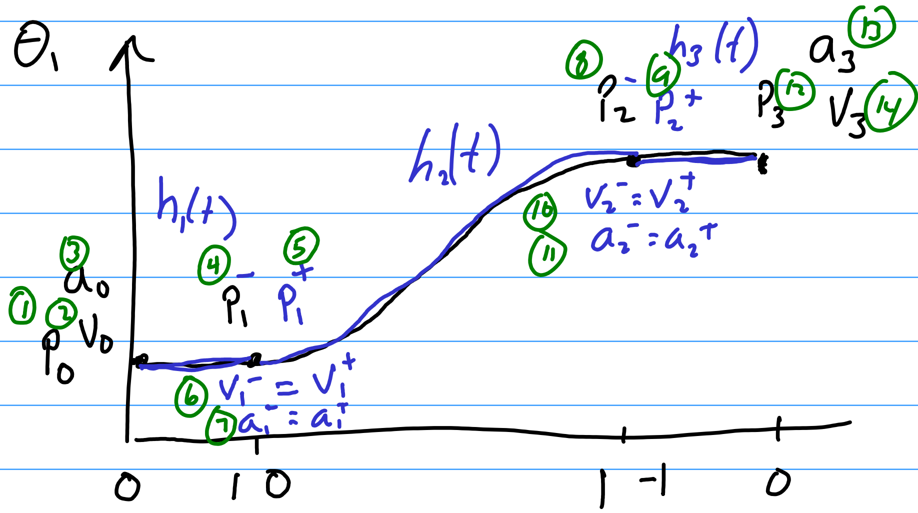
$v_1^- = v_1^+$) $\dot{h}_1(t_1) = \dot{h}_2(t_1) \Rightarrow 4a_{14}t_1^3 + 3a_{13}t_1^2 + 2a_{12}t_1 + a_{11} = 3a_{23}t_1^2 + 2a_{22}t_1 + a_{21}$

$a_1^- = a_1^+$| $\ddot{h}_1(t_1) = \ddot{h}_2(t_1) \Rightarrow 12a_{14}t_1^2 + 6a_{13}t_1 + 2a_{12} = 6a_{23}t_1 + 2a_{22}$

etc. 14 linear equations
14 unknowns -

Similar steps for 3-5-3

Can Use "Normalized time"



Velocity $\dot{h}_1(t)$

real velocity = $\frac{\dot{h}_1(t)}{t_1 - t_0}$

acceleration

real acceleration = $\frac{\ddot{h}_1(t)}{(t_1 - t_0)^2}$

p_0| $h_1(0) = p_0$

v_0) $\dot{h}_1(0)/t_1 - t_0 = v_0$ $\qquad a_{11}/t_1 - t_0 = v_0$

a_0) $\ddot{h}_1(0)/(t_1 - t_0)^2 = a_0$

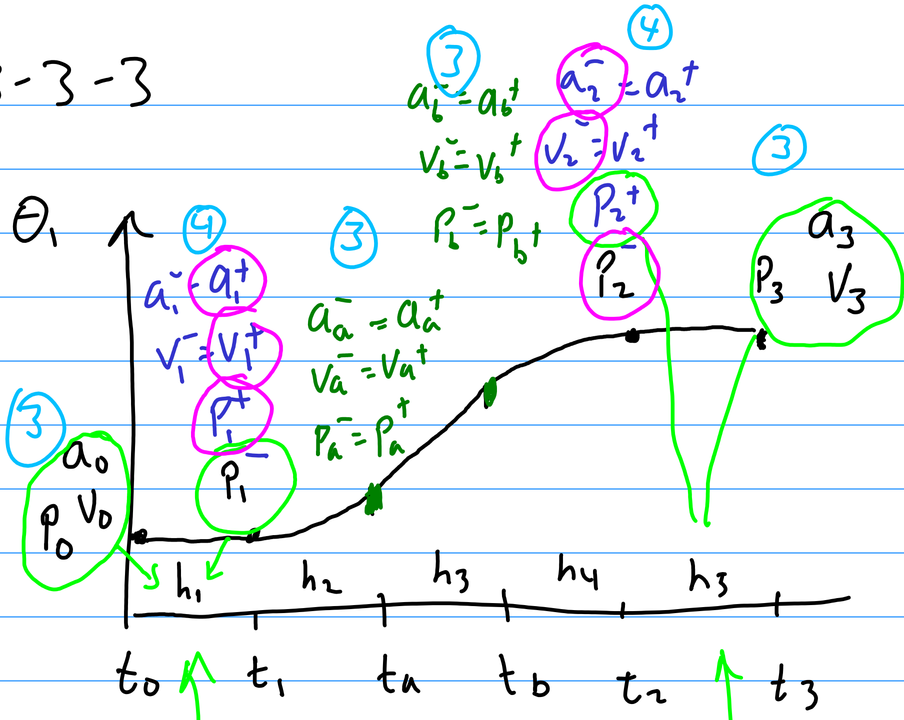
p_1^-) $h_1(1) = p_1$ $\qquad a_{14} + a_{13} + a_{12} + a_{11} + a_{10} = p_1$

p_1^+) $h_2(0) = p_1$ $\qquad a_{20} = p_1$

$v_1^- = v_1^+$| $\frac{\dot{h}_1(1)}{t_1 - t_0} = \frac{\dot{h}_2(0)}{t_2 - t_1}$

$a_1^- = a_1^+$| $\frac{\ddot{h}_1(1)}{(t_1 - t_0)^2} = \frac{\ddot{h}_2(0)}{(t_2 - t_1)^2}$

3-3-3-3-3



20 constraints

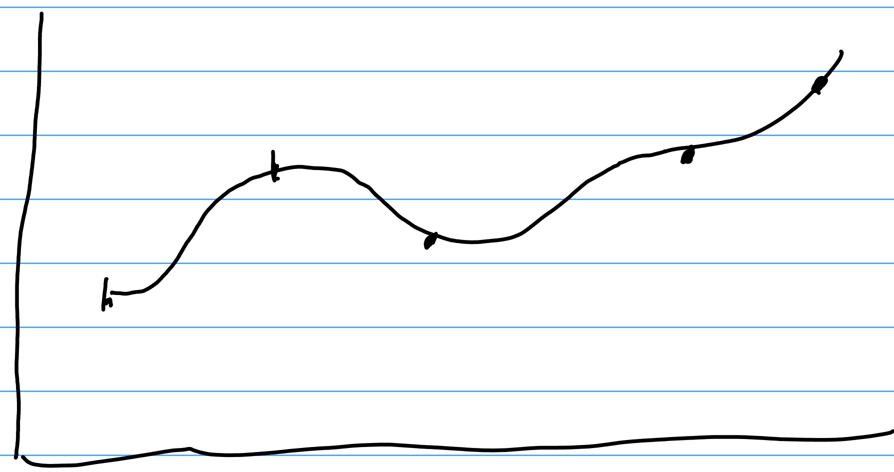
5 3rd order polynomials

solve for h_1 independently

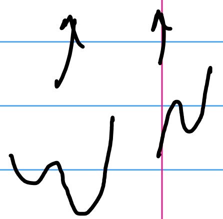
independent

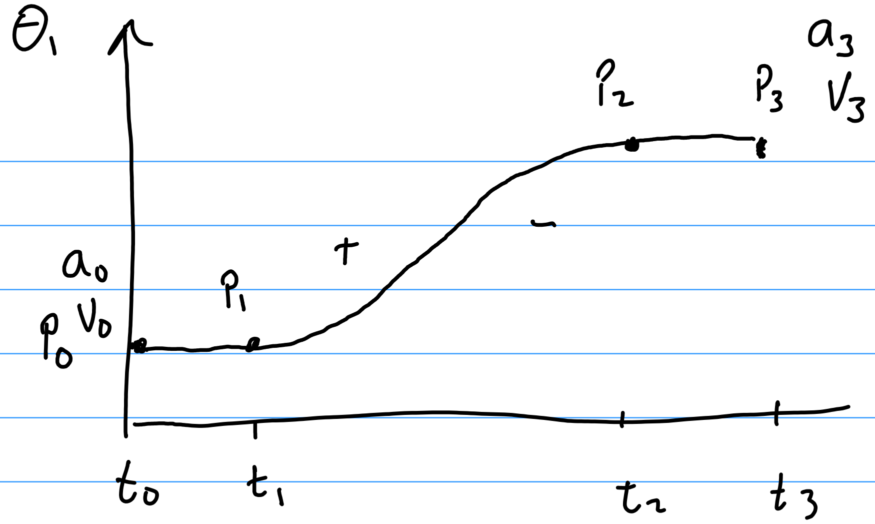
Cubic spline

eg. plot smooth curve between points



4-3-4





Concavity