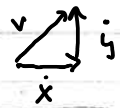
Mobile Robot Kinematics

Inverse Kinematics

Given $\bar{X}(t)$ $\binom{x}{y}$ find $\omega(t)$, $v(t)$

$\dot{\bar{X}}$ + $\ddot{\bar{X}}$ ↑ velocity

$\frac{d\theta}{dt}$ = how fast direction changes



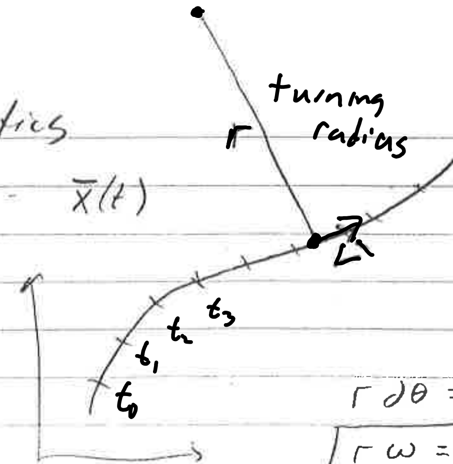
$$\boxed{V = \sqrt{\dot{x}^2 + \dot{y}^2}}$$

$$\theta = \tan^{-1} \frac{\dot{y}}{\dot{x}} = \text{current direction}$$

$$\omega = \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{\dot{y}}{\dot{x}}\right)^2}\left[\ddot{y}\,\dot{x}^{-1} + \dot{y}(-1)(\dot{x})^{-2}\ddot{x}\right]$$

$$\boxed{\omega = \frac{(\ddot{y}\dot{x} - \dot{y}\ddot{x})}{(\dot{x}^2+\dot{y}^2)}} = \frac{(\ddot{y}\dot{x} - \dot{y}\ddot{x})}{V^2}$$

note $\theta = \int \omega\, dt + \theta_0$



$r\, d\theta = d\,\text{dist}$

$\boxed{r\omega = V}$ ↑ radians/sec

↑ instantaneous turn radius

Forward Kinematics

Given $\theta(d)$ x_0 y_0 θ_0 and $d(t)$ or $v(t)$

Find $x(d)$, $y(d)$

Note $d(t) = \int V(t)dt + d_0$

$\frac{dx}{dd} = \cos\theta(d) \Rightarrow dx = dd \cdot \cos\theta(d)$

$\frac{dy}{dd} = \sin\theta(d) \Rightarrow dy = dd \cdot \sin\theta(d)$

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$$x(d) = d\cos\theta - \int d\, -\sin\theta \frac{d\theta}{dd} \cdot dd + x_0$$

$$\boxed{x(d) = d\cos\theta + \int d \sin\theta(d) \cdot \frac{d\theta}{dd} \cdot dd + x_0}$$

$$y(d) = d\sin\theta - \int d\cos\theta \frac{d\theta}{dd} \cdot dd + y_0$$

$$\boxed{y(d) = d\sin\theta - \int d\cos\theta(d) \frac{d\theta}{dd} \cdot dd + y_0}$$

Note: if $\frac{d\theta}{dd}$ = constant = W ⇝ $\theta = \theta_0 + Wd$ $0 = d_0 \Leftrightarrow \theta_0$, $d_1 \Leftrightarrow \theta_1$

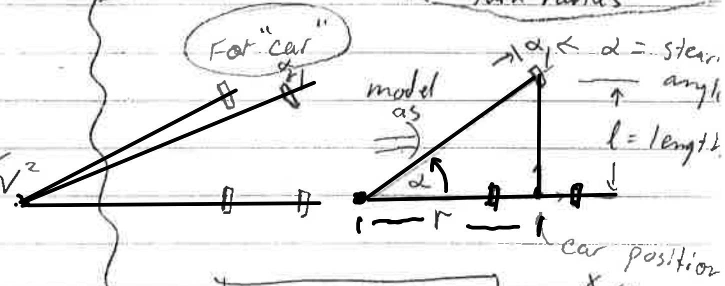
$$X_1 = d_1\cos\theta_1 + W\left[-d\cos(\theta_0+Wd)\frac{1}{\omega}\Big|_{d_0}^{d_1} - \int_{d_0}^{d_1} -\cos(\theta_0+\omega d)\frac{1}{\omega}\right] + X_0$$

$$= d_1\cos\theta_1 - (d_1\cos\theta_1 - \cancel{d_0\cos\theta_0}) + \frac{1}{\omega}\sin(\theta_0+\omega d)\Big|_{d_0}^{d_1} + X_0$$

$$\boxed{X_1 = (\sin\theta_1 - \sin\theta_0)\frac{1}{\omega} + X_0}$$

similarly

$$\boxed{y_1 = \ldots = (\cos\theta_0 - \cos\theta_1)\frac{1}{\omega} + y_0}$$



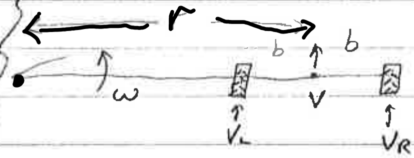
$$\boxed{\tan\alpha = \frac{l}{r}}$$

α = steering angle

l = length

r = turn radius

For "magog"



$$\boxed{\begin{aligned} V_L &= (r-b)\omega \\ V_R &= (r+b)\omega \end{aligned} \qquad r\omega = \frac{V_L+V_R}{2} \qquad b\omega = \frac{V_R-V_L}{2}}$$

$$d = \frac{D_R+D_L}{2} + d_0$$

$$\theta = \frac{D_R-D_L}{2b} + \theta_0$$

$$r = b \cdot \frac{V_R+V_L}{V_R-V_L}$$

$$\omega = \frac{V_R-V_L}{2b}$$

$$V = \frac{V_R+V_L}{2}$$

$D_R = \int V_R\, dt$ = dist R

$D_L = \int V_L\, dt$ = dist L