1. (12 pts) The circuit below shows a simple common-source amplifier with a source-degeneration resistor. Use a small-signal analysis to find the low-frequency gains $A_o = \frac{v_{out}}{v_{in}}$ and $A_1 = \frac{v_1}{v_{in}}$. Give your answer in terms of $R_{in}$, $R_S$, $R_D$, and any applicable small-signal parameters.

Assume that $\gamma = 0$, $\lambda = 0$, and $C_{DB} = C_{SB} = 0$

$C_{Depl} = \frac{C_{J0}}{(1 + V_R/V_o)^n}$, $C_{Diff} = \tau_F g_m$, $\omega_T = \frac{g_m}{C_{\mu} + C_{\pi}}$, $\omega_T = \frac{g_m}{C_{GD} + C_{GS}} \propto I_D^{1/2}$ or $V_{OV}$

$b_1 = \sum \tau_i$, $\tau_i = R_i C_i$, $\omega_H \approx \frac{1}{b_1}$, $Z_1 = \frac{Z}{1 - A}$, $Z_2 = \frac{Z}{1 - 1/A}$

Solution: The small signal model is illustrated on the right above. The controlled current source sets the current through $R_s$, and determines $v_1$:

$v_1 = (g_m v_{gs}) R_s = g_m R_s (v_{in} - v_1)$
$v_1 (1 + g_m R_s) = g_m R_s v_{in}$

$A_1 = \frac{v_1}{v_{in}} = \frac{g_m R_s}{1 + g_m R_s}$

Once $v_1$ is determined, the output voltage is found using

$v_{out} = -R_D (g_m v_{gs}) = -g_m R_D (v_{in} - v_1)$

Substituting the above value of $v_1$ gives

$v_{out} = -g_m R_D \left[ v_{in} - \frac{g_m R_s}{1 + g_m R_s} v_{in} \right] = -g_m R_D \left[ 1 - \frac{g_m R_s}{1 + g_m R_s} \right] v_{in} = \frac{-g_m R_D}{1 + g_m R_s} v_{in}$

$A_o = \frac{v_{out}}{v_{in}} = \frac{-g_m R_D}{1 + g_m R_s}$
2. (a) (8 pts) For the circuit of problem 1, examine the impact of the parasitic capacitors $C_{GD}$ and $C_{GS}$. Use the Miller theorem to redraw the circuit showing the equivalent capacitors between the various circuit nodes and ground. Give your component values in terms of $C_{GD}$, $C_{GS}$, $A_o = v_{out}/v_{in}$ and $A_1 = v_1/v_{in}$.

![Circuit Diagram]

**Solution:** Miller’s theorem relates the equivalent capacitance to the gain between the nodes that the capacitor is tied to. Use $A_1$ to deal with $C_{GS}$, and $A_o$ for $C_{GD}$.

- $C_{in} = C_{GS}(1 - A_1) + C_{GD}(1 - A_o)$
- $C_o = C_{GD}(1 - 1/A_o)$
- $C_1 = C_{GS}(1 - 1/A_1)$

(b) (8 pts) If $R_S$ is chosen sufficiently large, and $R_D \gg R_S$, then the gains from problem 1 can be approximated as

$$A_o = \frac{v_{out}}{v_{in}} \approx -\frac{R_D}{R_S} \quad \quad A_1 = \frac{v_1}{v_{in}} \approx 1$$

Using these approximations, argue that the bandwidth of the amplifier is primarily determined by $C_{GD}$ (and not $C_{GS}$).

**Solution:** If $A_1 \approx 1$, then all of the above $C_{GS}$ terms vanish. Essentially, no current will flow through $C_{GS}$ if the two terminal voltages are equal, so it drops out of the circuit. Only the $C_{GD}$ term remains... and the contribution to the input capacitance is potentially large. The above reduce to:

- $C_{in} \approx C_{GD} \left(1 + \frac{R_D}{R_S}\right)$
- $C_o \approx C_{GD} \left(1 + \frac{R_S}{R_D}\right) \approx C_{GD}$
- $C_1 \approx 0$
3. (a) (12 pts) Considering only $C_{GD}$, and using the approximations given in problem 2b, use the open-circuit time constant method to give an approximate expression for the amplifier bandwidth.

Solution:

The open circuit time constants are found by scaling each capacitor by the (DC) resistance measured at the capacitor terminals. At the input, this resistance is just $R_{in}$. The resistance at the output node is $R_D$ in parallel with the resistance seen looking into the drain of the transistor. In this case $r_o = \infty$ is given, so the result is just $R_D$.

\[
\tau_1 = R_{in}C_{in} = R_{in}C_{GD} \left(1 + \frac{R_D}{R_S}\right)
\]

\[
\tau_2 = R_DC_o = R_DC_{GD} \left(1 + \frac{R_S}{R_D}\right) = (R_D + R_S)C_{GD}
\]

The -3 dB frequency is approximately the reciprocal of the sum of all time constants:

\[
\omega_H \approx \frac{1}{\tau_1 + \tau_2} = \frac{1}{R_{in} \left(1 + \frac{R_D}{R_S}\right) + R_D + R_S} \left[\frac{1}{C_{GD}}\right] \text{ (in rad/sec)}
\]

(b) (3 pts) Assume that $g_m = 10 \text{ mA/V}$, $R_D = 5 \text{ k}\Omega$, $R_{in} = 100 \text{ k}\Omega$, $C_{GD} = 1 \text{ pF}$. Evaluate the (approximate) amplifier gain, 3-dB bandwidth (in kHz), and gain-bandwidth product for $R_S = 500 \text{ \Omega}$.

Solution:

\[
A_o = \frac{R_D}{R_S} = 10
\]

\[
\omega_H \approx \frac{1}{[(100 \text{ k}\Omega)(11) + (5 \text{ k}\Omega) + (500 \text{ \Omega})(1 \text{ pF})]} = 904.6 \text{ krad/sec}
\]

\[
f_H = \frac{\omega_H}{2\pi} = 143.96 \text{ kHz}
\]

\[
\text{GBW} = 10 \cdot f_H = 1.44 \text{ MHz}
\]

(c) (3 pts) Repeat problem 3b for $R_S = 1 \text{ k}\Omega$.

Solution:

\[
A_o = \frac{R_D}{R_S} = 5
\]

\[
\omega_H \approx \frac{1}{[(100 \text{ k}\Omega)(6) + (5 \text{ k}\Omega) + (1 \text{ k}\Omega)(1 \text{ pF})]} = 1650 \text{ krad/sec}
\]

\[
f_H = \frac{\omega_H}{2\pi} = 262.6 \text{ kHz}
\]

\[
\text{GBW} = 5 \cdot f_H = 1.31 \text{ MHz}
\]
4. The circuit below shows a folded cascode CMOS amplifier utilizing a simple current source $M_2$, supplying current $2I$, and a cascoded current source ($M_4$, $M_5$) sinking current $I$. All transistors are saturated. Assume, for simplicity, that all transistors have equal small-signal parameters $g_m$ and $r_o$, and that $g_m r_o \gg 1$. Neglect the body effect, and neglect all parasitic capacitors.

(a) (18 pts) Give approximate expressions for all of the resistances indicated: $R_{o1}$, $R_{o2}$, $R_{o3}$, $R_{o4}$, $R_{o5}$, $R_{in3}$ and $R_{out}$. Give your answers in terms of $g_m$ and $r_o$.

Solution: Start with the easy cases:

$$R_{o1} = R_{o2} = R_{o5} = r_o$$

The impedance looking into the drains of $M_4$ and $M_3$ have an additional term given by the resistance attached at their source terminals, scaled up by their intrinsic gains:

$$R_{o4} = r_{o4} + (1 + g_{m4} r_{o4}) r_{o5} \approx g_m r_o^2$$

$$R_{o3} = r_{o3} + (1 + g_{m3} r_{o3})(R_{o1} \parallel R_{o2}) = r_o + (1 + g_{m4} r_{o})(r_o/2) \approx g_m r_o^2/2$$

The resistance into the source of $M_3$ is $1/g_{m3}$ plus additional term given by the resistance attached to the drain terminal, scaled down by the intrinsic gain of $M_5$:

$$R_{in3} = \frac{1}{g_{m3}} + \frac{R_{o4}}{1 + g_{m3} r_{o3}} \approx \frac{1}{g_m} + \frac{g_m r_o^2}{g_m r_o} = \frac{1}{g_m} + r_o \approx r_o$$

Finally,

$$R_{out} = R_{o3} \parallel R_{o4} \approx \frac{\left(\frac{g_m r_o^2}{2}\right) g_m r_o^2}{\frac{g_m r_o^2}{2} + g_m r_o^2} = \frac{g_m r_o^2}{3}$$
(b) (12 pts) Re-evaluate $R_{in3}$ for the case when the output node is shorted to ground, and use current division to find the short-circuit transconductance $G_m$. Show that $G_m \approx g_{m1}$.

**Solution:** For the output shorted, the resistance attached to the drain of $M_3$ changes to 0 (instead of $R_{o4}$), so

$$R_{in3} = \frac{1}{g_{m3}}$$

If it helps, redraw the small-signal model for the case in which the output is shorted to ground as shown below. (With some practice, you should get to the point where you can work directly from the original circuit diagram.)

The controlled current source output from $M_1$ is divided between $R_{o1} \parallel R_{o2} = r_o/2$ and $R_{in3} = 1/g_m$. The output current is the current through $R_{in3}$:

$$i_{out} = g_{m1}v_{in} \left( \frac{r_o/2}{r_o/2 + 1/g_{m3}} \right) = g_{m1} \left( \frac{g_m r_o}{g_m r_o + 2} \right) v_{in}$$

$$G_m = \left. \frac{i_{out}}{v_{in}} \right|_{short} = g_{m1} \left( \frac{g_m r_o}{g_m r_o + 2} \right) \approx g_{m1} \quad \text{(for } g_m r_o \text{ Large)}$$

(c) (6 pts) Evaluate the voltage gain $v_{out}/v_{in}$ for the case $g_m = 2 \text{ mA/V}$, and $r_o = 10 \text{ k}\Omega$.

**Solution:** Here, the intrinsic gain is $g_m r_o = 20$. The amplifier voltage gain is

$$\frac{v_{out}}{v_{in}} = -G_m R_{out}$$

$$\approx -\frac{g_m^2 r_o^2}{3} = -\frac{(20)^2}{3} = -\frac{400}{3}$$

$$= -133.3$$
5. (18 pts) Using $V_{DD} = 1.8$ V and a pair of MOSFETs, design the current sink (shown below) to provide an output current $i_O$ of 200 $\mu$A nominal value. To simplify matters, assume that the nominal value of the output current is obtained at $v_O \approx V_{GS}$. It is further required that the circuit operate for $v_O$ in the range of 0.20 V to $V_{DD}$ and that the change in $i_O$ over this range be limited to 3% of the nominal value of $i_O$. Find the required value of $R$ and the device dimensions of $M_2$. For the fabrication-process technology utilized, $\mu_n C_{ox} = 250$ $\mu$A/V$^2$, $V_A = 20$ V/$\mu$m, and $V_t = 0.6$ V.

Solution: The required current sink characteristic is shown on the right above. The length of $M_2$ is found using the required output impedance.

$$r_{o2} \geq \frac{1.8 - 0.2}{6.6 \mu A} = 1.6 \text{ V} \div 6 \mu A = 266.6 \text{ k}\Omega$$

$$\frac{(20 \text{ V/}\mu\text{m})L_2}{200 \mu A} \geq 266.6 \text{ k}\Omega$$

$$L_2 \geq 2.66 \mu \text{m}.$$  

(I'll use the minimum required device length $L_2 = 2.66 \mu \text{m}$.) Since the current source must operate at $v_o = 0.2$ V, the overdrive voltage of $M_2$ (and $M_1$) is set to 0.2 V, giving $V_G = 0.8$ V for both MOSFETS. The width of $M_2$ is set to give the desired 200 $\mu$A output current:

$$200 \mu A = \frac{250 \mu A/V^2}{2} \left( \frac{W_2}{L_2} \right) (0.2 \text{ V})^2$$

$$W_2 \div L_2 = 40$$

$$W_2 = 40 L_2 = 40(2.66 \mu \text{m}) = 106.4 \mu \text{m}.$$  

The current through $M_1$ will be one fourth of that in $M_2$ because of the different value of $W_1 \div L_1$. So the design should set $I_{REF} = 50 \mu A$. Using the known gate voltage of $V_G = 0.8$ V, the value of $R$ is determined:

$$R = \frac{1.8 - 0.8 \text{ V}}{50 \mu A} = 20 \text{ k}\Omega.$$