1. (20 pts) Complete the design of the difference amplifier circuit shown below. Neglect the body effect and channel-length modulation (γ = 0, λ = 0). Assume that \( k'_n = 400 \mu A/V^2 \) and \( V_{tn} = 0.4 \) V.

Design for an overdrive voltage of 0.3 V for all transistors, and formulate your design to give \( g_{m1} = g_{m2} = 1 \) mA/V.

(a) Determine all MOSFET size ratios

\[
\begin{array}{c|c|c|c|c|c}
\text{W/L} & M_1 & M_2 & M_3 & M_4 & M_5 \\
\hline
& 8.33 & 8.33 & 12.77 & 8.33 & 8.33
\end{array}
\]

(b) Select a value for \( R_D \) to maximize the available output voltage swing when \( v_{icm} = 1.5 \) V.

\[ R_D = 6.33 \, k\Omega \]

(c) Determine the input common-mode range for your design.

\[ 1 \, V < v_{icm} < 2.45 \, V \] (\( = 3 - (150 \, \mu A)R_D + 0.4 \))

Solution:

(a) Select the size of \( M_1 \) and \( M_2 \) to give the desired \( g_m \) at \( V_{OV} = 0.3 \) V.

\[ g_m = 1 \, \text{mA/V} = k' \left( \frac{W}{L} \right)_{1,2} V_{OV} \quad \text{\( V_{OV} = 0.3 \) V, \( k' = 400 \, \mu A/V^2 \)} \quad \left( \frac{W}{L} \right)_{1,2} = 8.33 \]

\( M_4 \) and \( M_5 \) are the same size, since they carry the same drain current at the same overdrive voltage as \( M_1 \) and \( M_2 \). The actual drain current is

\[ I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{400 \, \mu A/V^2}{2} (8.33)(0.3 \, \text{V})^2 = 150 \, \mu A \]

Since \( V_{GS} = V_i + V_{OV} = 0.7 \) V, the drain current in \( M_3 \) is determined by the 10 kΩ resistor, \( I_{D3} = (2.3 \, \text{V})/(10 \, \text{kΩ}) = 230 \, \mu A \). Given \( I_{D3} \) and \( V_{OV} \), the required size is determined

\[ 230 \, \mu A = \frac{400 \, \mu A/V^2}{2} \left( \frac{W}{L} \right)_{3} (0.3 \, \text{V})^2 \quad \longrightarrow \quad \left( \frac{W}{L} \right)_{3} = 12.77 \]

(b) For \( v_{icm} = 1.5 \) V, \( M_1 \) and \( M_2 \) remain saturated as long as the output voltages remain above \( 1.5 - V_i = 1.1 \) V. To get the maximum output swing, bias the output voltages midway between this value and the positive supply rail, at 2.05 V.

\[ R_D = \frac{3 - 2.05}{150 \, \mu A} = 6.33 \, k\Omega \]

(c) \( M_4 \) and \( M_5 \) become ohmic if \( v_{D4} \) or \( V_{D5} \) drop below \( V_{OV} = 0.3 \) V. Since \( V_{GS1} = V_{GS2} = V_i + V_{OV} = 0.7 \) V, this requires that \( v_{icm} > 0.3 + 0.7 = 1.0 \) V.

To keep \( M_1 \) and \( M_2 \) pinched off (for the 2.05 V output bias voltage), we need \( v_{icm} < 2.05 + V_i = 2.45 \) V.
2. (20 pts) The circuit of problem 1 is reproduced below, except that \( M_4 \) and \( M_5 \) have been replaced by non-ideal current sources, and values of \( R_D \) are given. (Note that in practice implementing \( R_D = 50 \, \text{k}\Omega \) requires using an active load... not allowed in problem 1.). Continue to ignore the body effect and channel-length modulation for \( M_1 \) and \( M_2 \), and assume \( g_{m1} = g_{m2} = 1 \, \text{mA/V} \). Also, you may assume that \( R_s << 100 \, \text{k}\Omega \). Define the differential input signal as \( v_1 - v_2 \), and take \( v_{o1} \) as the (single-ended) output signal.

(a) Draw the differential-mode half-circuit, write down the differential gain expression in terms of the value of \( R_S \). Select \( R_S \) so that the design has a differential gain of -10.

(b) Draw the common-mode half circuit, and calculate the common-mode gain and common-mode rejection ratio of your design.

Solution:

Part a:

Using an approximation \( 100 \, \text{k}\Omega \parallel (R_s/2) \approx R_s/2 \), the gain of the amplifier is

\[
\frac{v_{o1}}{v_d/2} = -\frac{g_{m1}(50 \, \text{k}\Omega)}{1 + g_{m1}(R_s/2)}
\]

\[
A_d = \frac{v_{o1}}{v_d} = \frac{1}{2} \left[ -\frac{(1 \, \text{mA/V})(50 \, \text{k}\Omega)}{1 + (1 \, \text{mA/V})(R_s/2)} \right]
\]

Setting the gain to \( A_d = -10 \) and solving for \( R_s \) gives

\[
R_s = 3 \, \text{k}\Omega
\]

Part b:

The gain of this common-source amplifier is

\[
A_{cm} = \frac{v_{o1}}{v_{icm}} = -\frac{g_{m1}(50 \, \text{k}\Omega)}{1 + g_{m1}(100 \, \text{k}\Omega)} = -\frac{-50}{100} = -0.495
\]

The common-mode rejection ratio is

\[
\text{CMRR} = \frac{A_d}{A_{cm}} = \frac{10}{-0.495} = 20.2 \quad (= 26.1 \, \text{dB})
\]
3. (10 pts) An implementation of the circuit of problem 2 is shown below, in which the resistance $R_s$ has been replaced by two transistors $M_6$ and $M_7$. Note that $M_6$ and $M_7$ share the same gate to source voltages as $M_1$ and $M_2$, but operate in the ohmic (triode) region. All transistors are fabricated in the same technology as for problem 1 ($k'_{n} = 400 \mu A/V^2$, $V_{tn} = 0.4 V$, $\gamma = 0$, $\lambda = 0$). Also assume that the design of problem 1 was successful, so that $M_1$ and $M_2$ are saturated with $V_{ov1} = V_{ov2} = 0.3 V$

(a) At $v_1 = v_2 = 1.5 V$, what DC voltages appear at the sources of $M_6$ and $M_7$? What current flows through $M_6$ and $M_7$?

(b) Find the size ratio $W/L$ for $M_6$ and $M_7$ which would be appropriate to implement the circuit of problem 2 with $R_S = 3 \text{ k}\Omega$.

Solution:

(a) The gate to source voltages of $M_1$ and $M_2$ are given as $V_i + V_{ov} = 0.4 + 0.3 = 0.7 V$. So the source voltages are

$$V_{S6} = V_{S7} = 1.5 V - 0.7 V = 0.8 V$$

Since there’s know voltage drop across $M_6$ and $M_7$, no current flows in these transistors ($M_6$ and $M_7$ are ohmic).

(b) To implement $R_s = 3 \text{ k}\Omega$, select $r_{DS} = 1.5 \text{ k}\Omega$ for each of $M_6$ and $M_7$.

$$r_{DS} = \frac{1}{k'(W/L)(V_{GS} - V_i)} = 1.5 \text{ k}\Omega$$

These transistors are biased at the same gate-to-source voltages as $M_1$ and $M_2$ ($V_{GS} - V_i = V_{OV} = 0.3 V$), so set

$$\left(\frac{W}{L}\right)_{6,7} = \frac{1}{(400 \mu A/V^2)(1.5 \text{ k}\Omega)(0.3 V)} = 5.55$$
4. For the two-stage amplifier shown below, neglect the body effect, and assume that

\[ V_t = 1 \text{ V} \quad k'_n \left( \frac{W}{L} \right)_{M1} = 2 \text{ mA/V}^2 \quad (V_A)_{M1} = (V_A)_{Q2} = 40 \text{ V} \quad \beta_{Q2} = 200 \]

\[ C_{gs} = 2 \text{ pF} \quad C_{gd} = 1 \text{ pF} \]
\[ C_{db} = 0 \quad C_{sb} = 0 \]
\[ C_{\mu} = 0.8 \text{ pF} \quad C_{\pi} = 9.5 \text{ pF} \]

The circuit should look familiar. It’s identical to a circuit from Test 1, except the the capacitor values have been specified. Hopefully, you were able to show on Test 1 that by selecting \( R_1 = 3 \text{ k}\Omega \) you obtain (at midband)

\[ I_{D1} = 103 \mu\text{A} \quad V_{OV1} = 0.316 \text{ V} \quad I_{C2} = 1 \text{ mA} \]
\[ \frac{v_{out}}{v_{in}} = -18.5 \text{ V/V} \quad \frac{v_1}{v_{in}} = 0.649 \quad \frac{v_{out}}{v_{1}} = -28.51 \text{ V/V} \]

Note: In the original exam, the value of \( V_{OV1} \) was incorrectly given as 0.613 V instead of 0.316 V. The solution here uses the corrected value (shown in red).

(a) \((10 \text{ pts})\) Use the Miller theorem to write down the (midband) amplifier input impedance \( R_{in} \), and calculate the amplifier gain \( \frac{v_{out}}{v_{sig}} \).

**Solution:** The feedback resistor \( R_F \) has been replaced by the Miller equivalent resistances in the schematic above (in red). The input impedance can be seen by inspection:

\[ R_{in} = \frac{10 \text{ M}\Omega}{19.5} = 512.8 \text{ k}\Omega \]

The system gain is

\[ \frac{v_{out}}{v_{sig}} = \frac{v_{in}}{v_{in}} \cdot \frac{v_{out}}{v_{out}} = \left( \frac{512.8}{512.8 + 100} \right) (18.5) = 15.54 \text{ V/V} \]

(b) \((20 \text{ pts})\) Estimate the lower 3-dB frequency of the amplifier.

**Solution:** Identify the “pole” frequencies for each bypass capacitor (use \( r_{o2} = 40 \text{ V/1 mA} = 40 \text{ k}\Omega \)):

\[ \omega_1 = \frac{1}{(0.1 \mu\text{F})(100 \text{ k}\Omega + 512 \text{ k}\Omega)} = 16.3 \text{ rad/sec} \quad f_1 = \omega_1/(2\pi) = 2.6 \text{ Hz} \]
\[ \omega_2 = \frac{1}{(1 \mu\text{F})(1 \text{ k}\Omega + 9.5 \text{ M}\Omega \parallel 3 \text{ k}\Omega \parallel r_{o2})} = 263 \text{ rad/sec} \quad f_2 = \omega_1/(2\pi) = 41.9 \text{ Hz} \]

The approximate lower 3-dB cutoff frequency is the sum of the pole frequencies:

\[ f_L \approx f_1 + f_2 = 44.5 \text{ Hz} \]
Estimate the upper 3-dB frequency of the amplifier.

**Solution:** To approximate \( f_H \), evaluate the time constants for the parasitic capacitors at nodes \( 1 \), \( 2 \), and \( 3 \). At each node, the Miller theorem is used to find the equivalent capacitance between the node and ground, and the time constant is evaluated by finding the equivalent impedance looking into the node.

**Node 1:**
\[
C_1 = C_{gd} + C_{gs}(1 - 0.649) = 1.7 \text{ pF}
\]
\[
\tau_1 = (1.7 \text{ pF})(100 \text{ k}\Omega \parallel 512.8 \text{ k}\Omega) = 142.2 \text{ ns}
\]

**Node 2:**
\[
C_2 = C_{gs}(1 - 1/694) + C_\pi + C_\mu(1 + 28.51) = 32 \text{ pF}
\]
\[
\tau_2 = (32 \text{ pF})(r_\pi \parallel 6.8 \text{ k}\Omega \parallel 1/g_{m1})
\]

The small signal parameters are
\[
r_\pi = \frac{\beta}{g_{m2}} = \frac{200 \cdot 25.8 \text{ mV}}{1 \text{ mA}} = 5.16 \text{ k}\Omega
\]
\[
g_{m1} = \frac{2I_D1}{V_{ov1}} = \frac{2(103 \mu\text{A})}{0.316 \text{ V}} = 651.9 \mu\text{A/V}.
\]

Substituting gives the time constant
\[
\tau_2 = 32 \text{ ns}
\]

**Node 3:**
\[
C_3 = C_\mu(1 + 1/28.51) = 0.83 \text{ pF}
\]
\[
\tau_3 = (0.83 \text{ pF})(1 \text{ k}\Omega \parallel 3 \text{ k}\Omega \parallel 9.5 \text{ M}\Omega \parallel r_{o2}) = 611 \text{ ps}
\]

The time constants are combined to approximate \( f_H \):
\[
\omega_H \approx \frac{1}{\tau_1 + \tau_2 + \tau_3} = \frac{1}{174.8 \text{ ns}} = 5.72 \text{ Mrad/sec}
\]
\[
f_H = \frac{\omega_H}{2\pi} \approx 910 \text{ kHz}
\]
\[ i_C = \alpha i_E = \beta i_B = I_s e^{v_{BE}/V_T} \left(1 + \frac{v_{CE}}{V_A} \right) \]

\[ V_T = \frac{kT}{q} \approx 25.8 \text{ mV at } T = 300 \text{ K} \quad \alpha = \frac{\beta}{\beta + 1} \]

\[ i_D = k' \frac{W}{L} \left( v_{GS} - V_t \right) v_{DS} - \frac{1}{2} v_{DS}^2 \]

\[ k' = \mu_n C_{ox} \]

\[ i_D = k' \frac{W}{L} (v_{GS} - V_t)^2 (1 + \lambda v_{DS}) \]

\[ \lambda = \frac{1}{V_A} = \frac{\lambda'}{L} \]

\[ V_t = V_{t0} + \gamma \left[ \sqrt{2\varphi_f + V_{SB}} - \sqrt{2\varphi_f} \right] \]

\[ g_m = \frac{I_C}{V_T} \]

\[ r_0 = \frac{V_A + |V_{CE}|}{I_C} \approx \frac{V_A}{I_C} \]

\[ r_\pi = \frac{\beta}{g_m} \]

\[ r_e = \frac{\alpha}{g_m} \]

\[ g_m = k' \frac{W}{L} V_{OV} = \frac{2I_D}{V_{OV}} = \sqrt{2k'(W/L)I_D} \]

\[ r_0 = \frac{V_A + |V_{DS}|}{I_D} \approx \frac{V_A}{I_D} \]

\[ g_{mb} = \frac{\gamma}{2 \sqrt{2\varphi_f + |V_{SB}|}} g_m \]
\[
A_{in} = \frac{-g_m(R_C || r_o) + r_o}{1 + g_m(R_C || r_o)R_E/R_C}
\]
\[
A_{is} = \beta
\]
\[
R_{in} = r_n + (\beta + 1)R_E
\]
\[
R_{out} = R_C || \{r_n + (1 + g_m r_o)(R_E || r_o)\}
\]

Use \( g'_m = g_m + g_{mb} = (1 + \chi)g_m \):

\[
R = \frac{1}{g_m} || r_0 + \frac{R_C}{1 + g_m r_0}
\]
\[
\approx r_e + \frac{R_C}{g_m r_0} \approx r_e
\]

With a base resistor, \( R_B \) is added to \( r_E \), and \( g_m \) is scaled by \( r_e/(r_e + R_B) \):

\[
R = r_0 + (1 + g_m r_0)(R_E || r_o)
\]

Use \( g'_m = g_m + g_{mb} = (1 + \chi)g_m \):

\[
R = \frac{1}{g_m} || \frac{R_C}{1 + g_m r_0}
\]
\[
\approx r_e + \frac{R_C}{g_m r_0} \approx r_e
\]

With a base resistor, \( R_B \) is added to \( r_E \), and \( g_m \) is scaled by \( r_e/(r_e + R_B) \):

\[
C_{Diff} = \tau_F g_m
\]
\[
\omega_T = \frac{g_m}{C_{GD} + C_{GS}} \propto I_D^{1/2} \text{ or } V_{OV}
\]
\[
a_1 = \sum \frac{1}{\tau_i} \approx \omega_L
\]
\[
b_1 = \sum \tau_i = \sum R_i C_i \approx \frac{1}{\omega_H}
\]
\[
Z_1 = \frac{Z}{1 - A}
\]
\[
Z_2 = \frac{Z}{1 - 1/A}
\]