1. Write down the inverse $z$-transforms of the following. Simplify your answers so that they are real:

(a) $\frac{z^3}{z - 1.1}$ \quad $|z| > 1.1$

(b) $z^3 - 3z^{-5}$

(c) $\frac{-z}{(z - 1/3)(z - 3)}$ \quad $\frac{1}{3} < |z| < 3$

(d) $\frac{5z^2 + 3z}{z^2 - 2z + 2}$ \quad $|z| > \sqrt{2}$
2. A causal 2nd order system is known to have transfer function

\[
H(z) = \frac{2z^2 + \frac{3}{2}z - 1}{z^2 + \frac{1}{4}z - \frac{3}{16}} = \frac{2z^2 + \frac{3}{2}z - 1}{(z - \frac{1}{2})(z + \frac{3}{4})}
\]

(a) Find the impulse response of the system.
(b) Give a difference equation that could be used to implement the system.
(c) Is this system stable? (Justify)
(d) Draw a block diagram of the Direct-Form II implementation of the system.
(e) An input signal \( x(n) = u(n) \) is applied to the system. Write down the functional form of the system output \( y(n) \). Give your answer in the form \( y(n) = Aq_1(n) + Bq_2(n) + \ldots \), where the functions \( g_i() \) are given, but the coefficients \( A, B, C \) etc. are left undetermined.
(f) For the input of problem 2e, find \( \lim_{n \to \infty} y(n) \).
3. Find the transfer function $H(z)$ (including the R.O.C.) and difference equation of a causal, stable, first-order IIR system with real coefficients that satisfies the following constraints:

(a) DC gain of 1.0,
(b) Gain of 0.0 at $f = \frac{1}{2}$.
(c) Gain magnitude of $-20$ dB at $f = \frac{1}{4}$. 
4. A software receiver is to be designed which is capable of demodulating any signal located within the FM broadcast band (88 MHz < |F| < 108 MHz). In order to illustrate the sampling process, assume that the receiver input signal has (continuous-time) spectrum $S_c(F)$ as shown below.

The signal is to be sampled using a sampling frequency of 120 Msps, as illustrated below.

(a) The (continuous-time) bandpass filter $H_c(F)$ must pass the entire FM broadcast band, and must reject any other continuous-time frequencies which will alias onto the FM band when the signal is sampled. Specify the required stop-band for this filter. Make the transition bands (between the passband and stopband edges) as large as possible.

(b) Draw and carefully label the DTFT of the discrete-time signal $y(n)$ for $-1 < f < 1$. (Do not bother including the “undesired signal” portion of the signal.)
General Definitions

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$x(n) = \frac{1}{2\pi j} \oint X(z)z^{n-1}dz$$

$$X^+(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \quad x(n) = \frac{1}{2\pi j} \oint X(z)zn^{-1}dz$$

$$x(n) = \frac{1}{2\pi j} \oint X^+(z)zn^{-1}dz, \quad n \geq 0$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}$$

Linear Systems

$$y(n) = x(n)*h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$Y(z) = X(z)H(z) \quad Y(\omega) = X(\omega)H(\omega)$$

Transform Properties

— Double-Sided Z-Transforms —

$$a_1x_1(n) + a_2x_2(n) \Leftrightarrow a_1X_1(z) + a_2X_2(z)$$

$$x(n-k) \Leftrightarrow z^{-k}X(z)$$

$$a^n x(n) \Leftrightarrow X(z/a)$$

$$x(n)*h(n) \Leftrightarrow X(z)H(z)$$

— Single-Sided Z-Transforms —

$$x(n-k) \Leftrightarrow z^{-k}X^+(z) + x(-k) + x(-k+1)z^{-1} + \ldots + x(-1)z^{-k+1}$$

$$x(n + k) \Leftrightarrow z^kX^+(z) - x(k-1)z - x(k-2)z^2 - \ldots - x(0)z^k$$

— DTFT —

$$e^{j\omega n}x(n) \Leftrightarrow X(\omega)$$

$$x_1(n)x_2(n) \Leftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda)X_2(\omega-\lambda)d\lambda$$

Transform Pairs

— Double-Sided Z-Transforms —

$$\delta(n) \Leftrightarrow 1$$

$$a^n u(n) \Leftrightarrow z^n z^{-1}, \quad |z| > |a|$$

$$-a^n u(-n-1) \Leftrightarrow \frac{z^n}{z^{-a}}, \quad |z| < |a|$$

$$a^n \cos(\omega_0 n) u(n) \Leftrightarrow \frac{1 - az^{-1} \cos(\omega_0)}{1 - 2az^{-1} \cos(\omega_0) + a^2 z^{-2}}, \quad |z| > |a|$$

$$a^n \sin(\omega_0 n) u(n) \Leftrightarrow \frac{az^{-1} \sin(\omega_0)}{1 - 2az^{-1} \cos(\omega_0) + a^2 z^{-2}}, \quad |z| > |a|$$

— DTFT (“in the limit”) —

$$u(n) \Leftrightarrow \frac{e^{j\omega} + 1}{e^{j\omega} - 1} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega - 2\pi k)$$

$$e^{j\omega_0 n} \Leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 - 2\pi k)$$

Sampling Theorem

$$X(F/F_s) = F_s \sum_{k=-\infty}^{\infty} X_c(F - kF_s)$$

$$f = F/F_s + k \quad X(f) = F_s \sum_{k=-\infty}^{\infty} X_c(Fs(f - k))$$

$$x_c(t) = \sum_{n=-\infty}^{\infty} x(n) \sin(\pi(t - nT_s)/T_s) / \pi(t - nT_s)/T_s$$

Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1 - x}, \quad |x| < 1$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^{x}, \quad |x| < \infty$$

General

(Standard Trig Identities will be furnished on request)

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$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$