

# ECE 214 – Electrical Circuits Lab

## Lecture 7

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# Announcements

- Will be out of town next Monday and Tuesday. For lecture I am switching with ECP214, so you will have ECP214 Tuesday at 8am and ECE214 Thursday at 8am.
- This will complicate Tues and Wed labs a bit, I'll do my best to make sure you get the material in time.



# Lab #7 – RLC Circuits



# Inductors



- Impedance:  $Z_L = j\omega L$
- Current lags voltage by  $90^\circ$  in sinusoid
- Store energy in magnetic field.
- Measured in Henries
- Chokes – block AC while letting DC pass



# Inductors – Estimating Value

- For a torroid, rectangular cross section, air filled

Magnetic field inside:  $B = \frac{\mu_0 I}{2\pi r}$

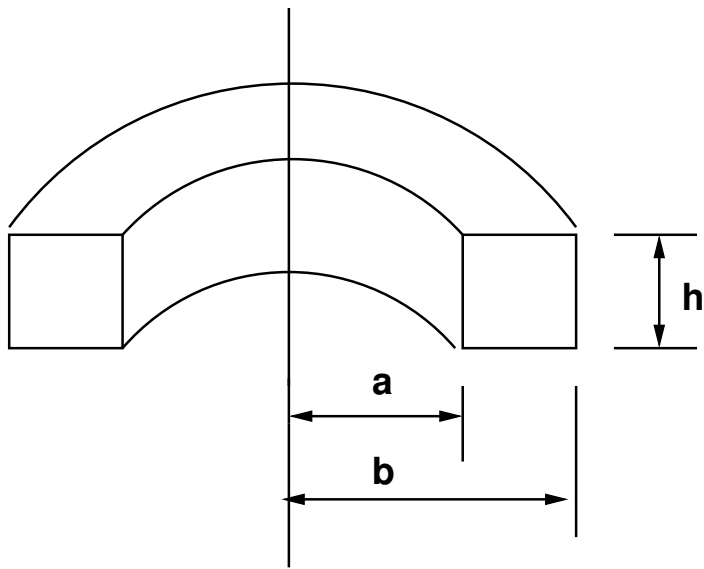
Magnetic flux through each loop turn:

$$\Phi_B = \int_a^b B h dr = \frac{\mu_0 I N h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I N h}{2\pi} \ln \frac{b}{a}$$

$$\text{Inductance: } L = \frac{N \Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}$$

- $N$  = number of turns  
 $h$  = height  
 $b$  = outside radius  
 $a$  = inside radius





- For ferrite core, multiply by ferrite's  $\mu_r$  which is anywhere from 16 to 640. Can you see why they use ferrite cores?
- Saturation – when have an iron core at high currents can saturate, causing big decrease in inductance.



# Calculating the Series Resistance using Q

- ESR = equivalent series resistance, Q = Quality Factor
- For an Inductor,  $Q = \frac{2\pi fL}{R_S}$
- For a Capacitor,  $Q = \frac{1}{D} = \frac{R_S}{2\pi fC}$   
D = Dissipation Factor,



# RLC Circuits – Resonance

- When impedance is at minimum (and non-imaginary)  
So when capacitance and inductance balance out.
- $\omega_0 = \frac{1}{\sqrt{LC}}$
- The capacitor and inductor bounce the current back and forth between each other.  $V$  on the Capacitor and  $V$  on the inductor are the same, but 180 degrees out of phase.
- Damping: whether circuit will resonate naturally. Depends on resistance. One that will be underdamped,



those that won't are overdamped.

attenuation = alpha, measured in nepers per second  
(decibels, but natural log)

$$\text{Damping factor} = \zeta = \frac{\alpha}{\omega_0}$$

Zeta = 1 is critical damping, just before oscillating.

- Resonance can be used as a filter. Bandwidth  $\delta\omega = 2\alpha$
- Q factor, describes resonators. Peak energy stored in circuit divided by the average energy dissipated in it per radian at resonance. Low Q circuits are damped and lossy; high Q circuits are underdamped. Q is related



to bandwidth; low Q circuits are wide band and high Q circuits are narrow band.

$$Q = \frac{\omega_0}{\delta\omega}$$

$$\text{Series RLC, } Q = \frac{1}{\omega_0 RC} = \frac{\omega_0 L}{R}$$



# RLC Circuits – Series

- $\alpha = \frac{R}{2L}$  and  $\omega_0 = \frac{1}{\sqrt{LC}}$



# RLC Circuits – Applications

- Tuning, with variable capacitor
- Filters
- Oscillator (where R is very small)
- Voltage Multiplier – If R is small,  $V_L = \frac{V}{R}\omega_0 L$   
 $\frac{V_L}{V} = Q$



# RLC Circuits – Analysis

- Step function: three cases.

$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad \zeta = \frac{\alpha}{\omega_0}$$

- Overdamped ( $\alpha > \omega_0$ ) ( $\zeta > 1$ )

Decaying exponential (after brief pulse up)  $v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

- Critically Damped ( $\alpha = \omega_0$ ) ( $\zeta = 1$ )

Quickly decaying exponential

$$v_C(t) = (A_1 + A_2 t) e^{-\alpha t}$$



– Underdamped ( $\alpha < \omega_0$ ) ( $\zeta < 1$ )

Ringling sinusoidal that gradually decays

$$v_C(t) = [A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)] e^{-\alpha t}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

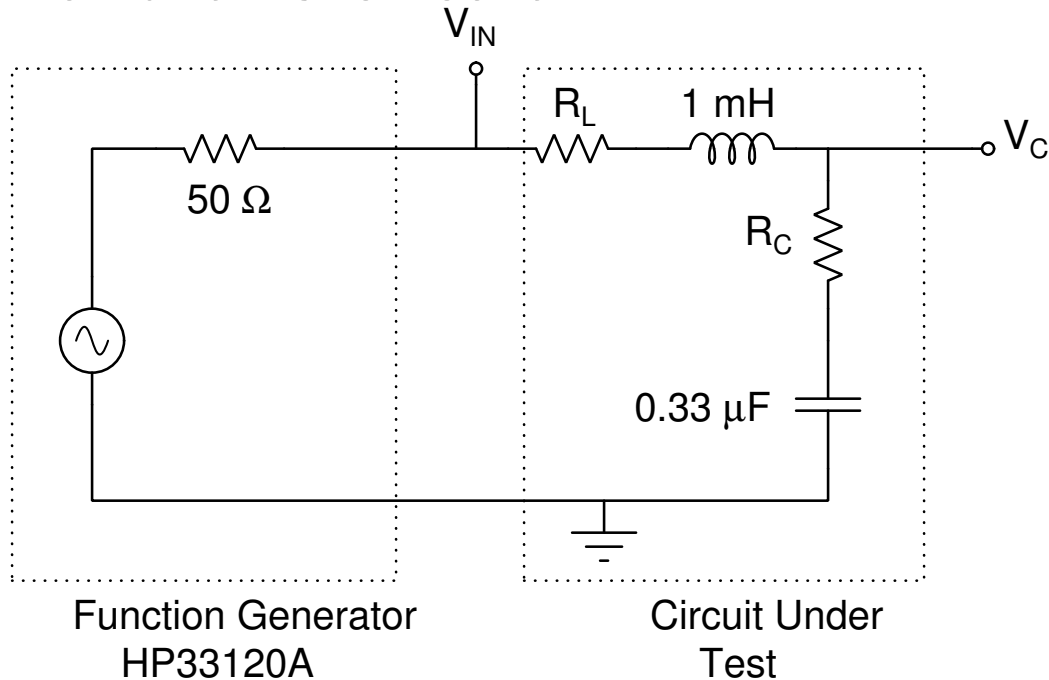
• Sinusoidal

$$v_C(t) = \frac{V_{in}}{LC} \frac{1}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + (\frac{R}{L}\omega)^2}} \cos(\omega t - \tan^{-1}(\frac{\frac{R\omega}{L}}{\frac{1}{LC} - \omega^2}))$$



# Lab Notes

- Build this circuit:



- You will need to do some micro-cap in-lab, be sure one



of your lab group has a laptop capable of doing this.

- Advanced equipment usage. As always you can check the relevant manuals, but here are some spoilers.
  - Hooking the trigger from the function generator to the external trigger on the scope. May be necessary?  
Co-ax cables in the closet.
  - Using the function generator in single-pulse mode:  
Set the signal up the way you want: (50ms, square, amplitude, offset).  
Press “shift” then “burst”



Can set number of repeats in burst with “shift” “<”  
Now press “single” (trig) key and it will send a single pulse.

- Using the scope in to capture a single shot:  
First it's best to get the scope set up with the proper scaling settings to begin with.  
Select Slope/Coupling and set the trigger level  
Mode / Single  
Press the Run button. It will collect on trigger but only a single shot.

- Finding phase shift in lissajous mode;



45 degree line to right means 0 degrees  $\sin^{-1}(0/max)$   
oval with the right ratio is 45 degrees  $\sin^{-1}(\frac{\sqrt{2}}{2})$   
circle is 90 degrees  $\sin^{-1}(max/max)$

