aka Tabular Method

Quine-McCluskey Method

--A systematic way to minimize a function
--How you might do it with a computer
--Might help get a better handle on the process

Two steps:
1) Find all prime implicants (include don't-cares as if they are 1's)
2) Select a minimum set of prime implicants to cover the 1's (ignore don't cares)

Note:
1) To find the prime implicants use XY or XY* = X

Combine two terms if they differ in one variable

Procedure:

Step 1

1) List all the minterms (include don't cares) in binary
Group them according to the number of 1's

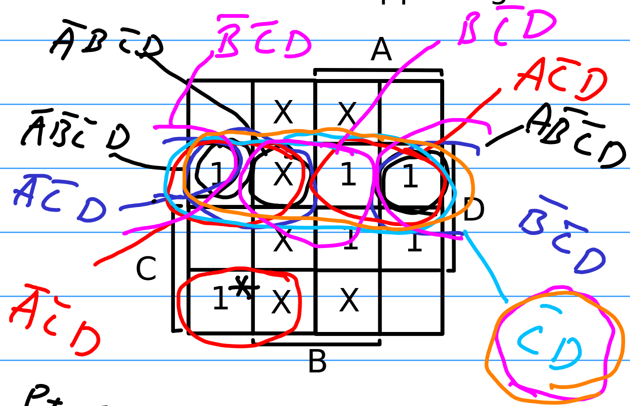
2) Combine those which differ in only one variable in the next column

3) Repeat (2) until no more can be combined

Step 1

Find all Prime Implicants

The K-map isn't part of the process, but we'll use it to show what is happening



Example:

f(A,B,C,D) = $\sum$ m(1,2,9,11,13,15) + d(4,5,6,7,12,14)

0001 0010 1001 0100 0101 1100 0110

all checked off
none are Prime implicants

a)

group 0
1 -0001 ✓ $A\bar{B}\bar{C}D$

group 1
2 0010 ✓
4 0100 ✓
5 -0101 ✓ $\bar{A}B\bar{C}D$

group 2
6 0110 ✓
9 1001 ✓
12 1100 ✓ $AB\bar{C}\bar{D}$

7 -0111 ✓
11 1011 ✓

group 3
13 1101 ✓
14 1110 ✓

group 4
15 1111 ✓

All implicants covering one minterm

b)

1,5 → 0-01 ✓ $\bar{A}\bar{C}D$
1,9 → -001 ✓ $\bar{B}\bar{C}D$
0-10
010- ✓
01-0 ✓
-100 ✓
01-1 ✓ 5,7
-101 ✓ 5,13 $B\bar{C}D$
011- ✓
-110 ✓
10-1 ✓
1-01 ✓ $A\bar{C}D$
110- ✓
11-0 ✓ 12,14
-111 ✓
1-11 ✓ 7,15
11-1 ✓ 11,15
111- ✓

PI 2,6

All implicants covering two minterms

c)

PI 1,5,9,13
--01 $\bar{C}D$
01--
-10-
-1-0
-1-1
-11-
1--1
11--

PI 9,11,13,15

Each entry is formed two ways

All implicants covering four minterms

d)

PI
-1--

Each entry is formed three ways

All implicants covering eight minterms

PI
2,6
1,5,9,13
9,11,13,15
4,5,6,7,12,13,14,15

Here we are trying each pair from adjacent groups
Combine if they differ in exactly one position
    Put them in the next column (keep in order)
    Check off the two that combined
    The way it is organized this means that dashes must match and
        1's in the upper group must have a 1 in that position below.
        The position that changes from 0 (above) to 1 (below) becomes
            a dash in the combination in the next column

All unchecked entries represent Prime Implicants

Step 2

Select a minimum set of
Prime Implicants to cover the 1's
(Ignore the don't-cares)

$$f = \sum m(1,2,9,11,13,15) + d(4,5,6,7,12,14)$$

0010
0110
0-10

Prime Implicant Chart

PI →

minterms

		1	2	9	11	13	15	
EPI	2,6		X					$\bar{A}C\bar{D}$
EPI	1,5,9,13	X		X		X		$\bar{C}D$
EPI	9,11,13,15			X	X	X	X	AD
	4,5,6,7,12,13,14,15					X	X	

$$f = \bar{A}C\bar{D} + \bar{C}D + AD$$

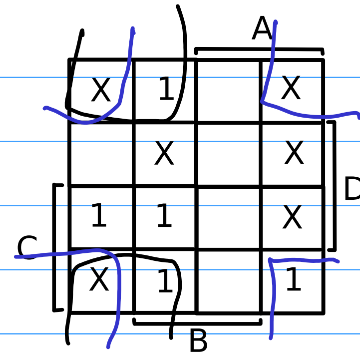
Identify Essential Prime Implicants -- any column with a single X means that product term MUST be used

Cross off the EPI and the minterms it covers

* Here the functions is covered by EPI

If there are remaining minterms, then find a minimal set to cover them (see next example)

Again, the K-map isn't part of the procdure



New Example

$$f = \sum m(3,4,6,7,10) + d(0,2,5,8,9,11)$$

Step 1

group 0	0000	00-0	0--0
		0-00	-0-0
		-000	
group 1	0010	001-	0-1-
	0100	0-10	-01-
	1000	-010	01--
		010-	10--
		01-0	
		100-	
		10-0	
group 2	0011	0-11	
	0101	-011	
	0110	01-1	
	1001	011-	
	1010	10-1	
		101-	
group 3	0111		
	1011		

Anything left unchecked is a prime implicant

Step 2

$$f = \sum m(3,4,6,7,10) + d(0,2,5,8,9,11)$$

Prime Implicant Chart

No Essential Prime Implicants

	3	4	6	7	10
0,2,4,6		X	X		
0,2,8,10					X
2,3,6,7	X		X	X	
2,3,10,11	X				X
4,5,6,7		X	X	X	
8,9,10,11					X

Select a minimum number of rows to that collectively have at least one X in each column

Here you can "reason" the minimal cover

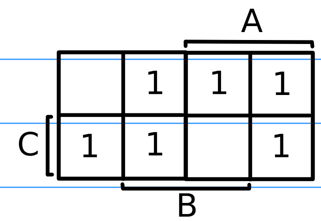
You can't cover all minterms with just one row

Try two rows. Note that each row has an equal "cost" (2-input AND)

Can you cover it in just two rows?

How about this one?

$$f = \sum m(1,2,3,4,5,6)$$



	1	2	3	4	5	6
1,3	X		X			
2,3		X	X			
2,6		X				X
4,6				X		X
4,5				X	X	
1,5	X				X	

Same problem, just rearranging the columns

	1	3	2	6	4	5
1,3	X	X				
2,3		X	X			
2,6			X	X		
4,6				X	X	
4,5					X	X
1,5	X					X

Petrick's method for chosing a minimal cover:

Previous problem

		3	4	6	7	10
A	0,2,4,6		X	X		
B	0,2,8,10					X
C	2,3,6,7	X		X	X	
D	2,3,10,11	X				X
E	4,5,6,7		X	X	X	
F	8,9,10,11					X

Label the rows. Now for

minterm 3 you must have (C + D)
minterm 4 you must have (A + E)
minterm 6 you must have (A + C + E)
minterm 7 you must have (C + E)
minterm 10 you must have (B + D + F)

To cover everything you will need the ANDing of all of these:

(C + D) (A + E) (A + C + E) (C + E) (B + D + F)   <---- Turn this POS into a SOP

(C + D) (A + E) (C + E) (B + D + F)   To make it easier, combine some terms first

(C + DE) (A + E) (B + D + F)

(AC + CE + ADE + DE) (B + D + F)

(AC + CE + DE) (B + D + F)   Now multiply it out to get SOP

(ABC + ACD + ACF + BCE + CDE + CEF + BDE + DE + DEF)   These are all possible ways to cover the function

This one is the one with the fewest product terms