ECE-343 Test 2: Mar 24, 2010
6:00-8:00pm, Closed Book
Name : SOLUTION

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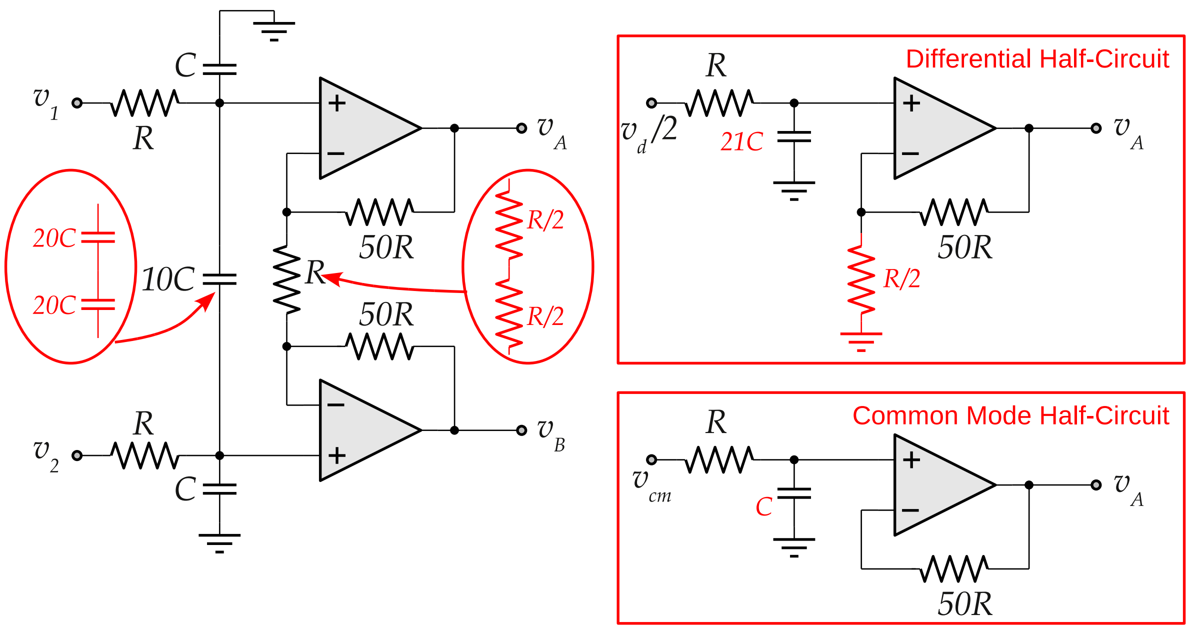
1. (25 pts) Assume that the operational amplifiers in the circuit below are ideal. Define the differential input signal as $v_d = v_1 - v_2$.

   (a) Draw the differential and common mode half-circuits, and use them to evaluate the differential and common-mode transfer functions for a single-ended output. (No other analysis technique allowed or graded.)

$$H_d(s) = \frac{v_A(s)}{v_d(s)} \qquad\qquad H_{cm}(s) = \frac{v_A(s)}{v_{cm}(s)}$$

   (b) For the single-ended output v_A, evaluate the *low-frequency* common-mode rejection ratio.



Solution:

(a) The Differential and Common mode half-circuits are found by replacing the $10C$ capacitor and the "joining" resistor by their equivalent series components as shown above. For the differential half-circuit, the node joining these series components is known to be at zero volts, so the circuit shown above-right can be examined. For the common-mode half-circuit, no current flows in these components, so the circuit shown below-right is appropriate.

The differential half circuit consists of a lowpass RC filter followed by a non-inverting amplifier. The differential transfer function is then

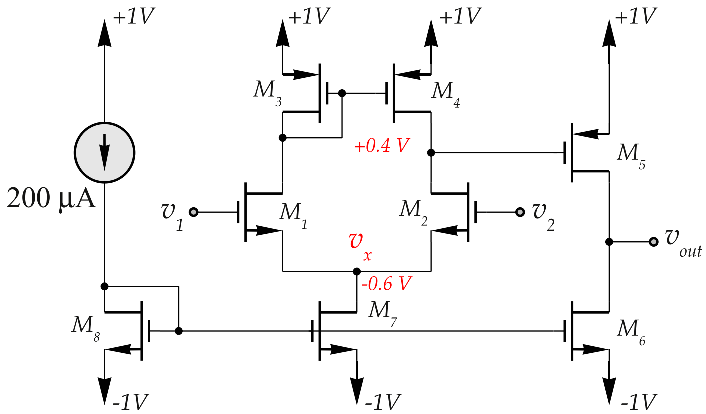
$$\frac{v_A}{v_d/2} = \left(\frac{1}{1+21RCs}\right)\left(1+\frac{50R}{R/2}\right) = \frac{101}{1+21RCs} \qquad \text{so, } H_d(s) = \frac{v_A(s)}{v_d(s)} = \frac{50.5}{1+21RCs}$$

The common mode half circuit is a lowpass filter (with different pole) followed by a unity gain voltage buffer, giving

$$H_{cm}(s) = \frac{v_A(s)}{v_{cm}(s)} = \frac{1}{1+RCs}$$

(b) $CMRR = 50.5/1 = 50.5$.

2. (45 pts) The two-stage CMOS amplifier shown below is fabricated in a $0.18 - \mu$m technology having $k_n' = 4k_p' = 400\ \mu\text{A/V}^2$, $V_{tn} = -V_{tp} = 0.4$ V. Neglect the body effect, assume that all nmos transistors have an Early voltage of $V_{An} = 6$ V, and pmos transistors have an Early voltage of $V_{Ap} = 5$ V. Define the differential input voltage as $v_{id} = v_1 - v_2$.



(a) Complete the design of the amplifier so that each of M_1, M_2, M_3, and M_4 have drain currents of 200 μA, and M_5 has a drain current of 1 mA. Design so that all transistors operate at an overdrive voltage of 0.2 V. Specify the W/L ratios for all transistors in the following table. (Please neglect channel-length modulation for this part of the problem.)

Transistor:	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
W/L:	25	25	100	100	500	125	50	25

Solution: Start with M_8, setting the drain current to 200 μA for a 0.2 V overdrive:

$$200\ \mu\text{A} = \frac{400\ \mu\text{A/V}^2}{2}\left(\frac{W}{L}\right)_8 (0.2)^2 \quad \longrightarrow \quad \left(\frac{W}{L}\right)_8 = \frac{1}{(0.2)^2} = 25$$

The other nmos transistors can be scaled from this result. M_1 and M_2 carry the same current as M_8; M_7 carries twice the current; and M_6 carries five times the current (all at the same overdrive voltage).

$$\left(\frac{W}{L}\right)_{1,2} = \left(\frac{W}{L}\right)_8 = 25 \qquad \left(\frac{W}{L}\right)_7 = 2\left(\frac{W}{L}\right)_8 = 50 \qquad \left(\frac{W}{L}\right)_6 = 5\left(\frac{W}{L}\right)_8 = 125$$

M_3 and M_4 each carry 200 μA. Using $k_p' = 100\ \mu\text{A/V}$ gives

$$200\ \mu\text{A} = \frac{100\ \mu\text{A/V}^2}{2}\left(\frac{W}{L}\right)_{3,4} (0.2)^2 \quad \longrightarrow \quad \left(\frac{W}{L}\right)_{3,4} = \frac{4}{(0.2)^2} = 100$$

M_5 has the same overdrive voltage as M_3 and M_4, and carries five times the current: $\left(\frac{W}{L}\right)_5 = 5\left(\frac{W}{L}\right)_{3,4} = 500$. (The DC bias voltages, assuming $V_{GS} = 0.6$ V for all transistors are labeled in the avove diagram.)

(b) Find the DC output voltage

Solution: Set the drain currents of M_5 and M_6 equal, taking into account the Early voltages:

$$\underbrace{\frac{k_n'}{2}\left(\frac{W}{L}\right)_6 (0.2)^2}_{1\text{ mA}}\left(1 + \frac{V_o + 1}{6\text{ V}}\right) = \underbrace{\frac{k_p'}{2}\left(\frac{W}{L}\right)_5 (0.2)^2}_{1\text{ mA}}\left(1 + \frac{1 - V_o}{5\text{ V}}\right)$$

$$\frac{V_o + 1}{6} = \frac{1 - V_o}{5} \quad \longrightarrow \quad \boxed{V_o = 0.0909\text{ V}}$$

(c) Find the input common mode range.

Solution: For M_7 to remain saturated, we need $v_x > -1 \text{ V} + V_{ov7} = -0.8$ V. For $v_1 = v_2 = v_{cm}$, we have $v_x = v_{cm} - V_{gs1} = v_{cm} - (V_t + V_{ov1}) = v_{cm} - 0.6$ V. So $v_{cm} > 0.2$ V is required.
Also, $V_{D1} = V_{D2} = 1 - V_{sg3} = 1 - 0.6 \text{ V} = 0.4$ V. M_1 will become ohmic if $v_{g1} > V_{d1} + V_t = 0.4 + 0.4 = 0.8$ V. This gives the upper limit on v_{cm}.

$$\boxed{-0.2 \text{ V} < v_{cm} < 0.8 \text{ V}}$$

(d) Find the allowable range of the output voltage.

Solution:

$$-1 + V_{ov6} < v_{out} < 1 - V_{ov5} \qquad \longrightarrow \qquad \boxed{-0.8 \text{ V} < v_{out} < 0.8 \text{ V}}$$

(e) Evaluate the differential gain of the amplifier v_{out}/v_{id}.

Solution: Multiply the differential gain of the first stage (differential amplifier) by the gain of the second stage (common source amplifier):

$$A_d = [g_{m1}(r_{o2} \parallel r_{o4})] \, [-g_{m5}(r_{o5} \parallel r_{o6})]$$

$$g_{m1} = \frac{2I_{D1}}{V_{ov1}} = \frac{2(200\ \mu\text{A})}{0.2 \text{ V}} = 2 \text{ mA/V} \qquad r_{o2} = \frac{6 \text{ V}}{200\ \mu\text{A}} = 30 \text{ k}\Omega \qquad r_{o4} = \frac{5 \text{ V}}{200\ \mu\text{A}} = 25 \text{ k}\Omega$$

$$g_{m5} = \frac{2(1 \text{ mA})}{0.2 \text{ V}} = 10 \text{ mA/V} \qquad r_{o6} = \frac{6 \text{ V}}{1 \text{ mA}} = 6 \text{ k}\Omega \qquad r_{o5} = \frac{5 \text{ V}}{1 \text{ mA}} = 5 \text{ k}\Omega$$

Both stages have a gain of 27.27:

$$\boxed{A_d = (27.27)(-27.27) = -743.8} \qquad (57.4 \text{ dB})$$

Solution: More accurate: Use V_{ds} in the calculation of r_o values. From the bias voltages of parts 2a and 2b,

$$g_{m1} = \frac{2(200\ \mu\text{A})}{0.2 \text{ V}} = 2 \text{ mA/V} \qquad r_{o2} = \frac{6 \text{ V} + 1 \text{ V}}{200\ \mu\text{A}} = 35 \text{ k}\Omega \qquad r_{o4} = \frac{5 \text{ V} + 0.6 \text{ V}}{200\ \mu\text{A}} = 28 \text{ k}\Omega$$

$$g_{m5} = \frac{2(1 \text{ mA})}{0.2 \text{ V}} = 10 \text{ mA/V} \qquad r_{o6} = \frac{6 \text{ V} + 1.09 \text{ V}}{1 \text{ mA}} = 7.09 \text{ k}\Omega \qquad r_{o5} = \frac{5 \text{ V} + 0.91 \text{ V}}{1 \text{ mA}} = 5.91 \text{ k}\Omega$$

$$\boxed{A_d = (31.11)(-32.23) = -1002.8} \qquad (60.0 \text{ dB})$$

(f) Evaluate the output impedance of the amplifier.

Solution:

$$R_{out} = r_{o5} \parallel r_{o6} = 2.722 \text{ k}\Omega \ \ \text{(or, more accurately, } 3.22 \text{ k}\Omega\text{)}$$

(g) Evaluate the common-mode gain of the amplifier.

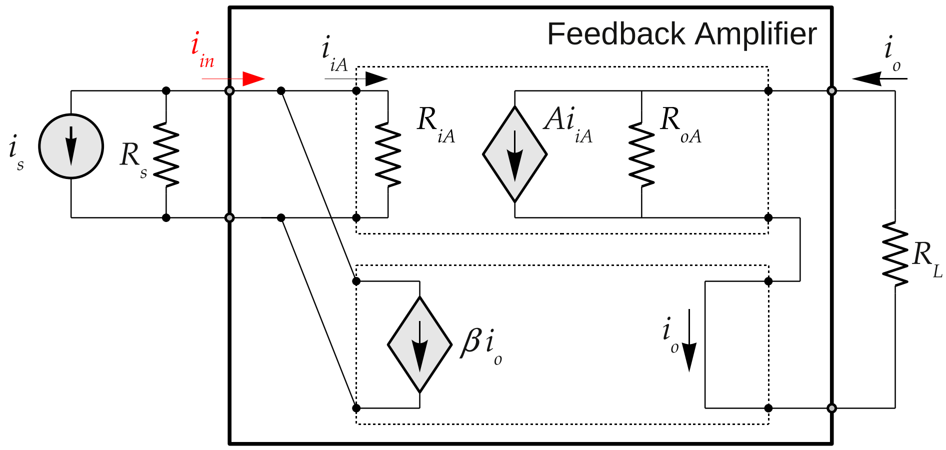
Solution: The common mode gain of the difference amplifier (1st stage) is

$$A_{cm1} = \frac{-1/g_{m3}}{2r_{o7}} = \frac{-1/(2 \text{ mA/V})}{2(6 \text{ V})/(400\ \mu\text{A})} = \frac{-500\Omega}{2(15 \text{ k}\Omega)} = \frac{-1}{60}$$

This is multiplied by the gain of the second stage to get the overall common mode gain:

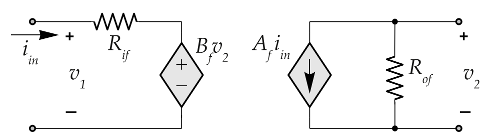
$$A_{cm} = \left(\frac{-1}{60}\right)(-27.27) = 0.4545 \qquad (-6.85 \text{ dB})$$

3. (30 pts) A current-mixing, current-sampling feedback technique is used in the design of a current amplifier as shown below.



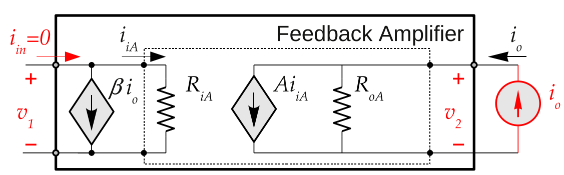
(a) Draw a two-port model which is appropriate for describing the "feedback amplifier" part of the circuit. The model should include the forward gain A_f (in A/A) of the closed loop system. It should also show the appropriate parameters R_{if}, R_{of}, and the reverse gain B_f.

Solution: The "h-parameters" provide a 2-port model with the appropriate current-controlled current source in the output port. The model is shown here with the parameters given in terms of the closed-loop circuit values:



(b) *DERIVE* the expression for B_f in terms of R_{iA}, R_{oA}, A, and β.

Solution: From the above 2-port (h-parameter) model, we know we can determine B_f by evaluating v_1/v_2 when port-1 is open ($i_{in} = 0$). The original closed-loop amplifier is redrawn below, with port-1 held open, and port-2 driven by a current source to set the value of i_o:



With i_o set, the port-1 voltages and currents fall out:

$$i_{iA} = -\beta i_o \qquad\qquad v_1 = R_{iA} i_{iA} = -\beta R_{iA} i_o$$

Since i_{iA} is determined, v_2 can be found

$$v_2 = (i_o - A i_{iA}) R_{oA} = (i_o + A\beta i_o) R_{oA} = (1 + A\beta) R_{oA} i_o$$

The end result is

$$\boxed{B_f = \left.\frac{v_1}{v_2}\right|_{i_{in}=0} = \frac{-\beta R_{iA}}{(1 + A\beta) R_{oA}}}$$

(c) Evaluate R_{if}, R_{of}, A_f, and B_f using the numerical values below. (No derivations needed, or graded.) Be sure to include units.

$$A = 250 \text{ A/A} \qquad \beta = 0.1 \text{ A/A} \qquad R_{iA} = 6 \text{ k}\Omega \qquad R_{oA} = 50 \text{ k}\Omega$$

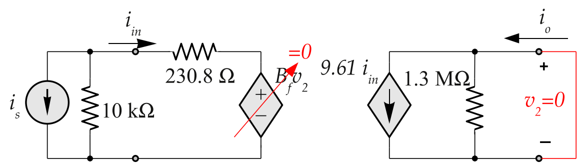
Solution: We have $1 + A\beta = 26$, so

$$A_f = \frac{250}{26} = 9.61 \text{ A/A} \qquad R_{if} = \frac{6 \text{ k}\Omega}{26} = 230.8 \ \Omega$$

$$B_f = \frac{-0.1(6 \text{ k}\Omega)}{26(50 \text{ k}\Omega)} = -4.61 \times 10^{-4} \text{ V/V} \qquad R_{of} = 26(50 \text{ k}\Omega) = 1.3 \text{ M}\Omega$$

(d) Find the system gain i_o/i_s when $R_L = 0$ and $R_s = 10$ kΩ (Use the values given in 3c).

Solution: The equivalent 2-port representation found in 3c is shown below.



For $R_L = 0$, the B_f source is not active, and current division gives the input current i_{in}. The gain is

$$\frac{i_o}{i_s} = -9.61 \left(\frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 230 \ \Omega} \right) = -9.61 \left(\frac{10 \text{ k}\Omega}{10.230 \text{ k}\Omega} \right) = 9.39 \text{ A/A}.$$

(e) Use the parameters given in 3c, except assume that $A \to \infty$. Find the gain A_f of the feedback amplifier.

Solution:

$$A_f = \frac{A}{1 + A\beta} \xrightarrow{A\to\infty} \frac{1}{\beta} = 10 \text{ A/A}$$

(f) What are the restrictions on the amplifier gain A so that A_f remains within 5% of the value you predicted in part 3e?

Solution:

$$\frac{A}{1 + A(0.1)} \geq 0.95(10) = 9.5 \qquad \longrightarrow \qquad \boxed{A \geq 190}$$