

1. Floating Point

(a) Convert the decimal value -7.5 to 32-bit floating point. Report the result in hex. For full credit, show your work.

(b) Convert the 32-bit floating point value $0xece27100$ to its decimal equivalent. For full credit, show your work.

2. Floating Point (Solution)

- (a) Convert the decimal value -7.5 to 32-bit floating point. Report the result in hex. For full credit, show your work.

32-bit floating point bit breakdown:

Sign=1 bit, Exponent=8 bits, Fraction=23 bits

Exponent Bias=127

-7.5 is negative, so **sign bit=1**

Take the absolute value (7.5) and Divide or multiply by 2 until it is greater than one but less than two. This is getting the number into the binary `1.fraction` expected by the format.

- $7.5/2 = 3.75$
- $3.75/2 = 1.875$
- So 7.5 is 1.875×2^2

(This is equivalent to converting 7.5 to binary, which is 111.1, and noting you have to shift the radix point two to the left to get it to be in `1.fraction` format. The divide/multiply by two in decimal algorithm gives you the same result and can be easier to do especially for large numbers).

Since we are multiplying by 2^2 this means the exponent is 2, but remember we have to adjust by the bias. So the exponent field= $2+127$ **exponent=129**

Finally we need to calculate the fraction part, 0.875. The textbook describes an algorithm where you repeatedly take the fractional part and multiply by 2 until you hit 1. Note that worse case you might never hit 1 (repeating decimal pattern) and in that case stop after 23 bits for single precision.

- $.875*2 = 1.75$, integer part is 1 (1)
- $.75 *2 = 1.5$, integer part is 1 (1)
- $.5*2 = 1$, integer part is 1 (1)

So fractional part is **111**.

You can verify this by remembering that 0.111 binary is $2^{-1} + 2^{-2} + 2^{-3} = .5 + .25 + .125 = .875$

Now we need to put the bits in the proper positions

Sign	Exponent+Bias	Fraction
(negative)	(129)	
1	1000 0001	1110 0000 0000 0000 0000 000
1100 0000	1111 0000 0000 0000 0000 0000	
C 0	F 0 0 0 0 0	0

The end result is **0xc0f00000**

- (b) Convert the 32-bit floating point value 0xece27100 to its decimal equivalent. For full credit, show your work.

First break things up into binary 32-bits

```
E      C      E      2      7      1      0      0
1110 1100 1110 0010 0111 0001 0000 0000
```

Remember that for 32-bit, Sign=1 bit, Exponent=8 bits, Fractional part is 23 bits.

```
Sign  Exponent      Fraction
1      1101 1001      1100 0100 1110 0010 0000 000
```

A sign bit of 1 means the result will be **negative**

The exponent is 0xd9, or 217 decimal. Remember to subtract off the bias (127), $217 - 127 = 90$ so the **exponent is 90**.

The rest is 1.fraction, so in our case = 1.1100010011100010

The fractional part is $\frac{1}{2} + \frac{1}{4} + \frac{1}{64} + \frac{1}{512} + \frac{1}{1024} + \frac{1}{2048} + \frac{1}{32768}$
 = 0.76907349

So the final floating point value is:

$$\begin{aligned}
 &= -1^{\text{signbit}} \times 1.\text{fraction} \times 2^{\text{exponent}-\text{bias}} \\
 &= -1 \times 1.76907349 \times 2^{90} \\
 &= -2.190 \times 10^{27}
 \end{aligned}$$

So the result is approximately $-2.190 * 10^{27}$