

# Binary

There are 10 kinds of people in the world  
those that understand binary and those who don't

## Place value system

Base 10

$$182 = 1 \times 10^2 + 8 \times 10^1 + 2 \times 10^0$$

Base 2

$$10110110 \Rightarrow 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$\begin{matrix} & 7 & & 6 & & 5 & & 4 & & 3 & & 2 & & 1 & & 0 \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ & 128 & & 64 & & 32 & & 16 & & 8 & & 4 & & 2 & & 1 \end{matrix}$

Binary  
→ decimal

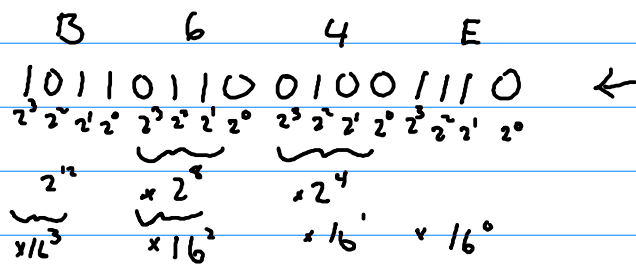
Decimal  
to binary

182	10110110	
↓	↖	8-bits ← default
182 - 128 = 54		16
54 - 32 = 22		32
22 - 16 = 6		
6 - 4 = 2		
2 - 2 = 0		

2   182	Remainder	
2   91	0	10110110
2   45	1	
2   22	1	
2   11	0	
2   5	1	
2   2	1	
2   1	0	
0	1	

↑  
Read  
Upwards

Hexadecimal AKA Hex base 16



$$B \times 16^3 + 6 \times 16^2 + 4 \times 16^1 + E \times 16^0$$

Hex  
Decimal  
Binary

0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

How about negative numbers?

1) Sign magnitude

-88                      +88 ⇒ 01011000  
 -88                      -88    11011000

2) Ones complement

+88 01011000  
 -88 10100111

Complement  
toggle

3) Two's complement

+88 01011000  
 10100111  
 +1  
 -88 10101000

negate?  
0 → 1  
1 → 0

With all three systems

most significant bit

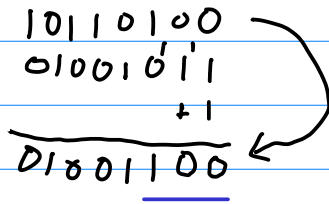
MSB = 0 → positive - same in each  
 = 1 → Negative

-88 = 10101000  
 01010111  
 +1  
 10101000

"taking the 2's complement"  
 ⇒ multiply by -1

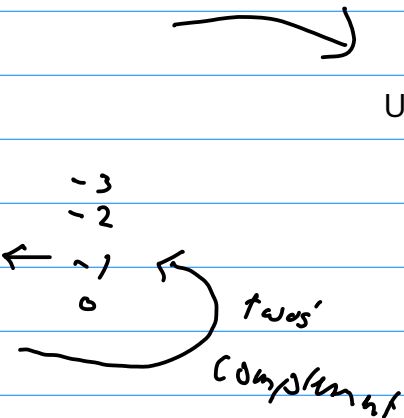
# Two's complement

short cut

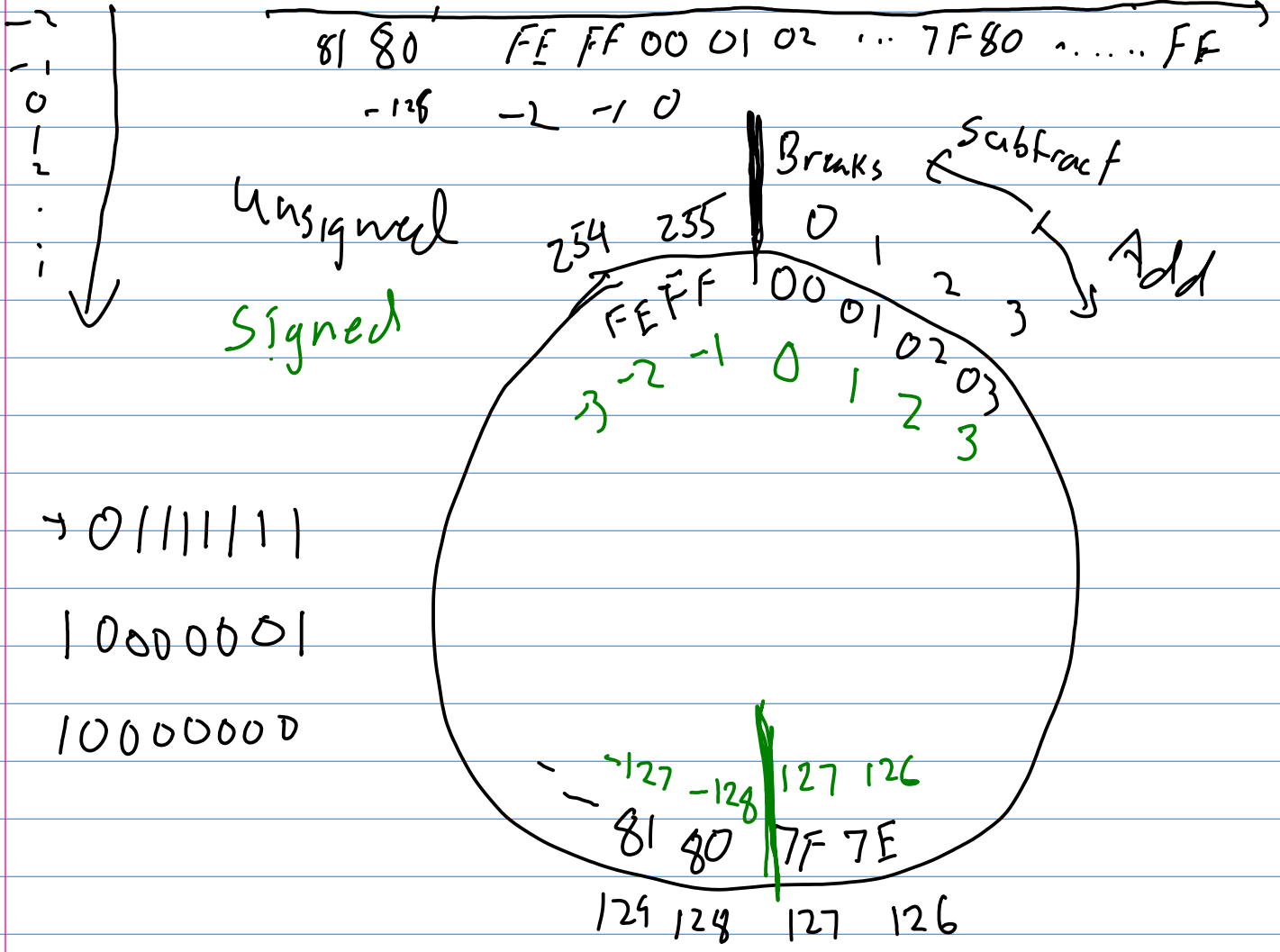


Copy bits from right to left, to and including the first "1" then complement the rest

Unsigned	Binary	HEX
251	11111011	FB
252	11111100	FC
253	11111101	FD
254	11111110	FE
255	11111111	FF
0	00000000	00
1	00000001	01
2	00000010	02
3	00000011	03
4	00000100	04
5	00000101	05
6	00000110	06
125	01111101	7D
126	01111110	7E
127	01111111	7F
128	10000000	80
129	10000001	81
251	11111011	FB
252	11111100	FC
253	11111101	FD
254	11111110	FE
255	11111111	FF
0	00000000	00
1	00000001	01



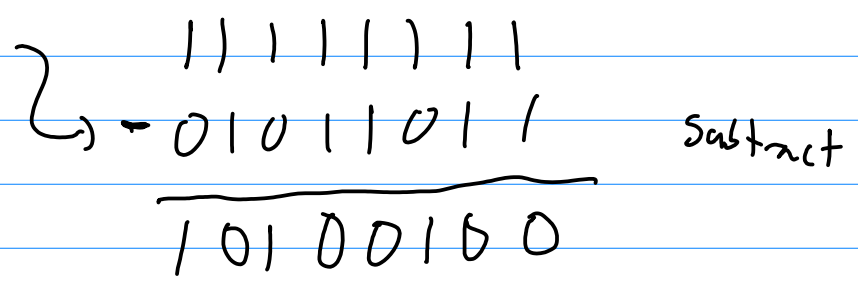
Unsigned	Binary	Signed
251	11111011	-5
252	11111100	-4
253	11111101	-3
254	11111110	-2
255	11111111	-1
0	00000000	0
1	00000001	1
2	00000010	2
3	00000011	3
4	00000100	4
5	00000101	5
6	00000110	6
125	01111101	125
126	01111110	126
127	01111111	127
128	10000000	-128
129	10000001	-127
251	11111011	-5
252	11111100	-4
253	11111101	-3
254	11111110	-2
255	11111111	-1
0	00000000	0
1	00000001	1



Ones complement

Subtract from

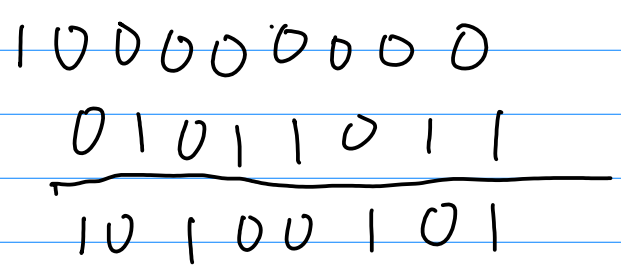
$$2^n - 1$$



twos complement

Subtract from

$$2^n$$

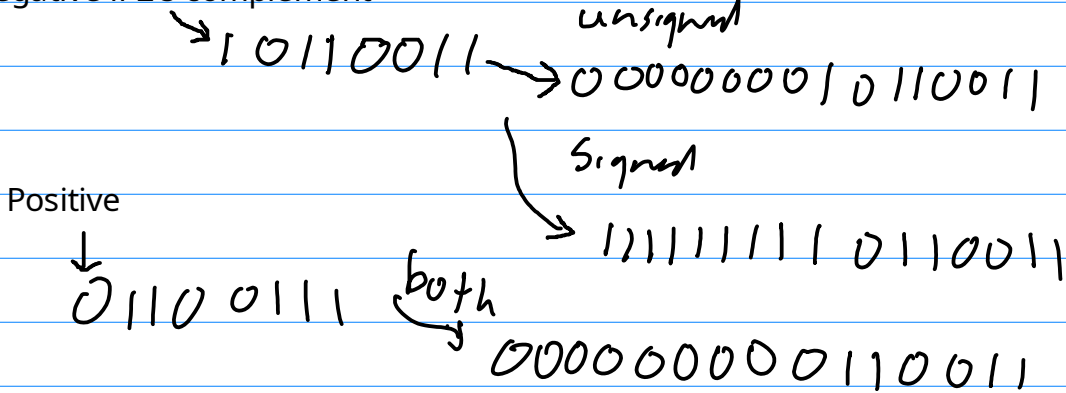


# Sign extension

eg. 8 bits  $\rightarrow$  16 bits

unsigned - add leading 0's  
Signed - "sign extend"

Negative if 2's complement



unsigned		Signed	
90	01011010	90	10
59	+ 00111011	59	"
149	$\leftarrow 10010101 \rightarrow$	-107	

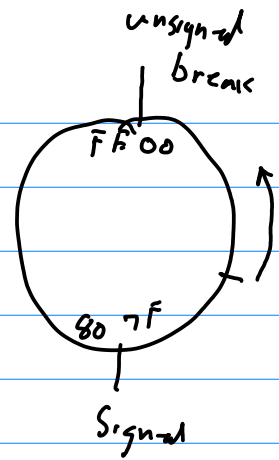
$\uparrow$  correct                       $\uparrow$  wrong

218	11011010	-38	
59	00111011	59	
21	00010101	21	

$\uparrow$  wrong                       $\uparrow$  correct

Subtraction

unsigned	$\begin{array}{r} 90 \\ 59 \\ \hline 31 \end{array}$	$\begin{array}{r} 01'0'0'1'0 \\ 01'0'0'0'1'0 \\ - 00111011 \\ \hline 00011111 \end{array}$	signed	$\begin{array}{r} 90 \\ - 59 \\ \hline 31 \end{array}$
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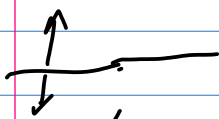
$$\begin{array}{r} 183 \\ - 675 \end{array}$$

multiply by powers of 2

$$\begin{array}{l} 0000011 \Rightarrow 3 \\ \times 2 \\ \hline 0000110 \\ \times 4 \\ \hline 0001100 \end{array}$$

$$\begin{array}{r} 133 \times 10 \\ 1330 \\ \downarrow \\ 13300 \end{array}$$

to multiply by  $2^n$  shift left by  $n$  places  
& fill in zeros



to divide by  $2^n$  shift to right  $n$  places

eg  $10110111$  divided by  $2^3$

$$\begin{array}{r} 10110111 \\ \underline{00010110} \end{array}$$

$$\begin{array}{r} 189 \div 10 \\ 18 \end{array}$$

If unsigned fill in zeros

$$01101001 / 2^3$$

If signed & positive fill in zeros  
& negative fill in ones

$$\begin{array}{r} 0001101 \\ 10101111 / 2^3 \\ 11110101 \end{array}$$

29. 11111111 ← -1  
 11111110 ← -2 / 2'  
 11111111 ← -1

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multiply

```

  10101101
  00110101
  -----
  10101101
  00000000
  10101101
  00000000
  10101101
  10101101
  -----
  1000111010001
  
```

101  
↑

11  
↓