$3 \times 3 \cdot 3 \times 1 \Rightarrow 3 \cdot 1$

# Translation + Rotation

$$\Rightarrow \bar{P}_{xyz} = {}^{xyz}_{uvw}\bar{R}\,\bar{P}_{uvw} + \bar{t}_{uvw} \quad \longleftrightarrow \quad \overset{3\times1}{\begin{pmatrix} \bar{P}_{xyz} \\ 1 \end{pmatrix}} = \underbrace{\begin{pmatrix} \bar{R} & \bar{t}_{uvw} \\ 000 & 1 \end{pmatrix}}_{\text{not orthonormal}} \begin{pmatrix} \bar{P}_{uvw} \\ 1 \end{pmatrix}$$

$$\bar{R}\,P_{uvw} = P_{xyz} + (-\bar{t}_{uvw})$$

$$\bar{P}_{uvw} = \bar{R}^T(\bar{P}_{xyz} + -\bar{t}_{uvw}) \qquad \bar{R}^T = \bar{R}^{-1}$$

Inverse

$$\Rightarrow \bar{P}_{uvw} = \underset{\substack{\uparrow \\ \text{rotation part}}}{\bar{R}^T \bar{P}_{xyz}} + \underbrace{-\bar{R}^T \bar{t}_{uvw}}_{\text{translation part}}$$

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## Homogeneous coordinates

we use $w = 1$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} \quad \bar{P} \Rightarrow \hat{P} = \begin{pmatrix} wP_x \\ wP_y \\ wP_z \\ w \end{pmatrix} \quad \hat{P} = \begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix}$$

$$\hat{P}_{xyz} = \begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix} = \underset{4\times4}{\begin{pmatrix} \overset{3\times3}{\bar{R}} & \overset{3\times1}{\bar{t}_{uvw}} \\ \underset{1\times3}{000} & \underset{1}{1} \end{pmatrix}} \begin{pmatrix} P_u \\ P_v \\ P_w \\ 1 \end{pmatrix} \quad \begin{pmatrix} \bar{P}_{uvw} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \bar{P}_{xyz} \\ 1 \end{pmatrix} = \begin{pmatrix} \bar{R} & \bar{t}_{uvw} \\ 000 & 1 \end{pmatrix} \begin{pmatrix} \bar{P}_{uvw} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} & t_u \\ r_{21} & r_{22} & r_{23} & t_v \\ r_{31} & r_{32} & r_{33} & t_w \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{{}^{xyz}T_{uvw}} \begin{pmatrix} P_u \\ P_v \\ P_w \\ 1 \end{pmatrix}$$

Inverses

Inverse

$$\bar{P}_{uvw} = \bar{R}^T \bar{P}_{xyz} + \underbrace{-\bar{R}^T \bar{t}_{uvw}} \longrightarrow \begin{pmatrix} \bar{P}_{uvw} \\ 1 \end{pmatrix} = \begin{pmatrix} \bar{R}^T & -\bar{R}^T\bar{t} \\ 000 & 1 \end{pmatrix} \begin{pmatrix} \bar{P}_{xyz} \\ 1 \end{pmatrix}$$

$$\underset{3\times1}{\begin{pmatrix} t_u \\ t_v \\ t_w \end{pmatrix}} \underset{1\times3}{(000)}$$

$$\begin{pmatrix} \bar{R} & \bar{t} \\ 000 & 1 \end{pmatrix} \begin{pmatrix} \bar{R}^T & -\bar{R}^T t \\ (000) & 1 \end{pmatrix} = \begin{pmatrix} \overset{\bar{I}_{33}}{R R^T} + \bar{t} \cdot \overset{\text{3x3 matrix of zeros}}{(000)} & -\bar{R}\bar{R}^T\bar{t} + \bar{t} \cdot 1 \\ (000)\,R^T + 1 \cdot (000) & (000) - R^T\bar{t} + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} \bar{I}_{33} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ (000) & 1 \end{pmatrix} = \bar{I}_{44}$$

Given ${}^{B}\bar{T}_A$ and ${}^{C}\bar{T}_B$ what is ${}^{C}\bar{T}_A$

$$\hat{p}^B = \overset{4\times4}{{}^{B}\bar{T}_A}\;{}^{A}\hat{p} \qquad {}^{C}\hat{p} = {}^{C}\bar{T}_B\;{}^{B}\hat{p} \qquad {}^{C}\hat{p} = {}^{C}\bar{T}_A\;{}^{A}\hat{p}$$

$${}^{C}\hat{p} = {}^{C}\bar{T}_B\;{}^{B}\bar{T}_A\;{}^{A}\hat{p}$$

$${}^{robot}\bar{T}_{camera} \cdot {}^{camera}\bar{T}_{object} = {}^{robot}\bar{T}_{object}$$

$${}^{robot}\bar{T}_{obj2}$$

$${}^{obj2}\bar{T}_{object} = \underbrace{{}^{obj2}\bar{T}_{robot}}_{\uparrow \; \left({}^{robot}T_{obj2}\right)^{-1}}\;{}^{robot}\bar{T}_{camera}\;{}^{camera}T_{object}$$

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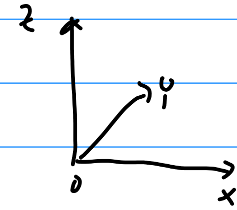
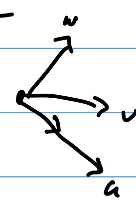
$$\begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix} = \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} p_u \\ p_v \\ p_w \\ 1 \end{pmatrix}$$




$(n_x, n_y, n_z)^T$ = direction of $\bar{i}_u$ wrt $oxyz$

$(s_x\ s_y\ s_z)^T$ = " " $\bar{j}_v$ wrt $oxyz$

$(a_x\ a_y\ a_z)^T$ = " " $\bar{k}_w$ wrt $oxyz$

$(t_x\ t_y\ t_z)$ = origin of $ouvw$ wrt $oxyz$

HW1 (6)

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$${}^{xyz}\bar{R}_{uvw} = {}^{xyz}\bar{R}_{world} \cdot {}^{world}\bar{R}_{uvw} = \begin{bmatrix} \bar{X}^T \\ \bar{Y}^T \\ \bar{Z}^T \end{bmatrix} \begin{bmatrix} \bar{U} & \bar{V} & \bar{W} \end{bmatrix} = \begin{bmatrix} \bar{X}^T\bar{U} & X^T\bar{V} & \bar{X}^T\bar{W} \\ \bar{Y}^T\bar{U} & \bar{Y}^T\bar{V} & Y^T\bar{W} \\ \bar{Z}^T\bar{U} & Z^T\bar{V} & Z^T\bar{W} \end{bmatrix}$$

Transpose

$${}^{world}\bar{R}_{uvw} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} = \begin{bmatrix} \bar{U} & \bar{V} & \bar{W} \end{bmatrix} \qquad {}^{world}\bar{R}_{xyz} = \begin{bmatrix} \bar{X} & \bar{Y} & \bar{Z} \end{bmatrix}$$

$$\begin{array}{ccc} {}^{xyz}\bar{p} & & {}^{xyz}\bar{T}_{uvw} \quad {}^{uvw}\bar{p} \\ \begin{pmatrix} 4 \\ -1 \\ 1 \\ 1 \end{pmatrix} & = & \begin{bmatrix} -1 & 0 & 0 & 5 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} \end{array}$$

$$= {}^{xyz}\bar{T}_{uvw} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$