Minimal SOP (Sums of Products)

Terms:

Literal - a single variable or its complement

Implicant - any single 1 or group of 1's that can be represented by a product term (Power of 2 in each direction). e.g., $\bar{A}BC$, $A\bar{D}$, B

Prime Implicant - implicants which cannot be combined to eliminate a variable. e.g. if both $A\bar{B}\bar{C}$ and $A\bar{B}C$ are implicants, they are not "Prime" as they can be combined using the combining theorem (aka "uniting theorem" in some texts) ---> $A\bar{B}\bar{C} + A\bar{B}C = A\bar{B}$

A minimal expression uses only Prime Implicants

Essential Prime Implicant (EPI) - a prime implicant which is the only PI to "cover" some (at least one) "1" on the K-map

Cover - a collection of Prime Implicants which all the 1's in the function (and no zeros, of course).

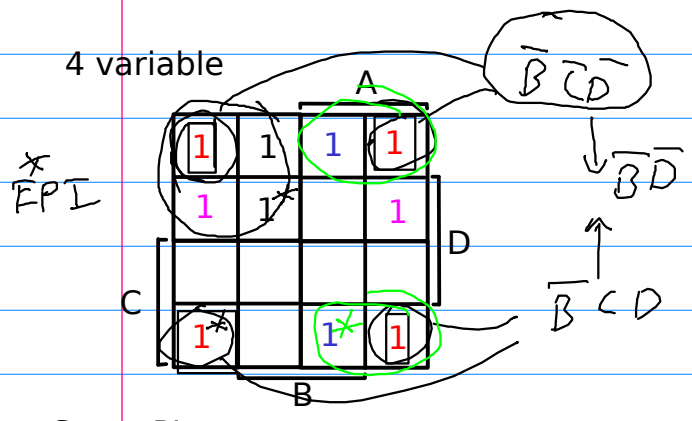
Cost - there are many measures. We will use # gates + # inputs
Inversions on individual inputs are not counted
Inversions on the output are counted

Minimization process:

1) Identify all the prime implicants
2) Identify all the Essential Prime Implicants (look at all the 1's in the K-map; if any are covered by just one PI, then that PI is an EPI. EPI must be used in a minimal SOP.
3) Select a minimal set of remaining PI to cover the remaining PI in the function

An important property of the K-map is that "adjacent" cells differ in only one variable and therefore the "combining theorem" can be used when they are both 1

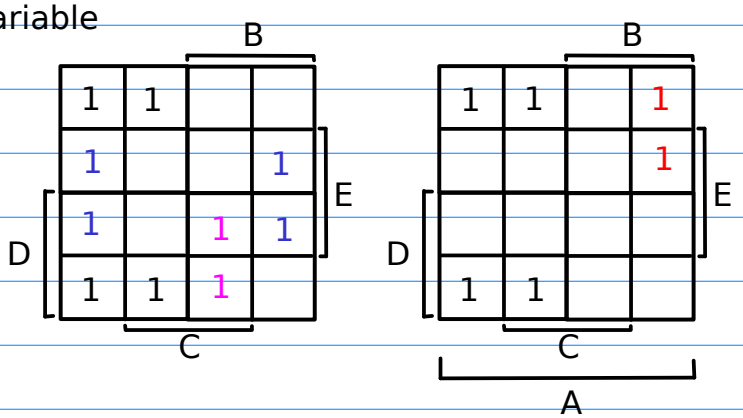
4 variable



Some PIs

$\bar{A}B\bar{C}$ $\bar{B}\bar{C}D$ $AB\bar{D}$ $\bar{B}\bar{D}$

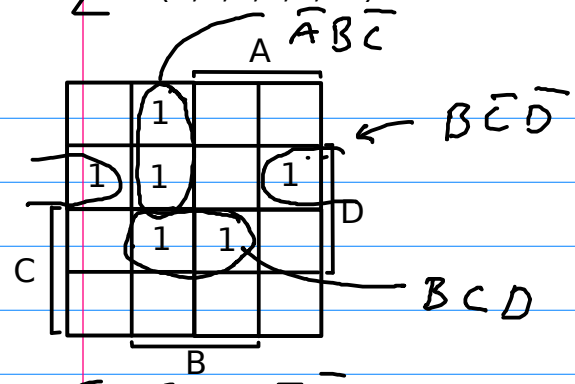
5 variable



Some PIs

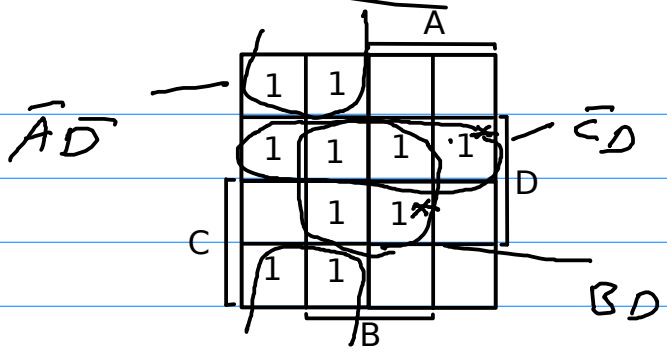
$\bar{B}\bar{E}$ $\bar{A}BCD$ $\bar{A}\bar{C}E$ $AB\bar{C}\bar{D}$

$f = \sum m(1,4,5,7,9,15)$

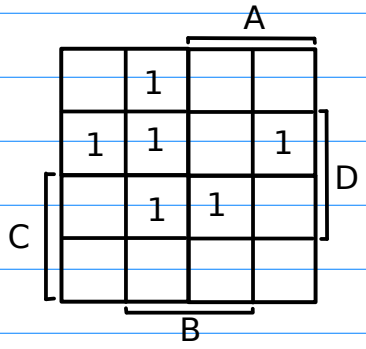


$f = \bar{A}B\bar{C} + B\bar{C}\bar{D} + BCD$

$f = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + AB\bar{C}\bar{D}$

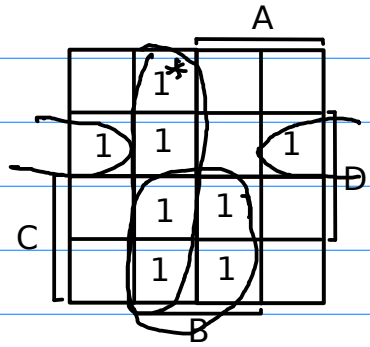


$f =$



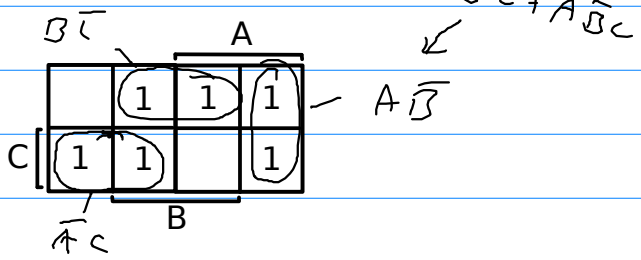
$f =$

Slightly modified



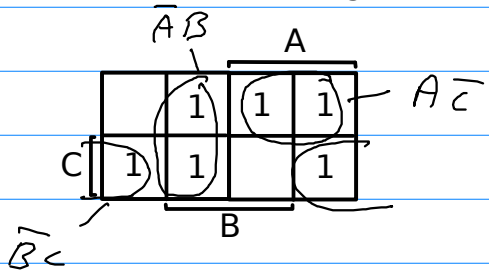
$f =$

We've seen this one before

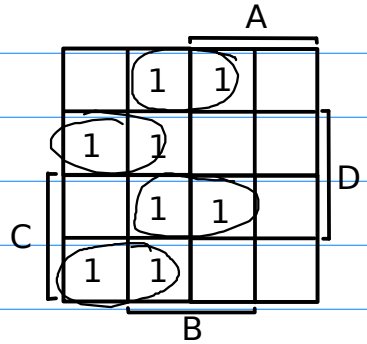


$f =$

Same function again



$f =$

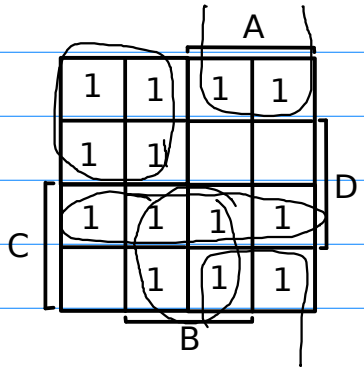


Don't automatically just choose the biggest groups

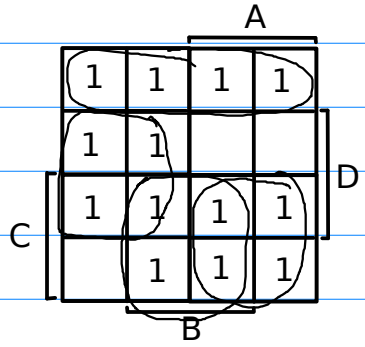
(This function happens to be completely covered by EPIs)

$f =$

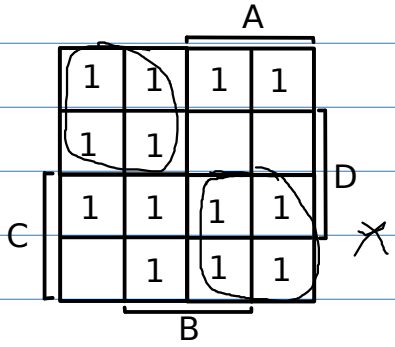
No EPI, so where do you start?



try
again



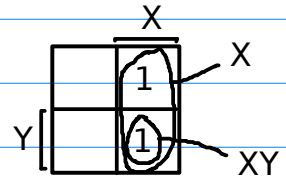
try
again



Note that there are 13 one's to cover. There are no PI that cover 8 ones; the biggest PI (actually all PI here) cover 4 ones, so a complete cover must include 4 PI (4 product terms). That is the best we can do

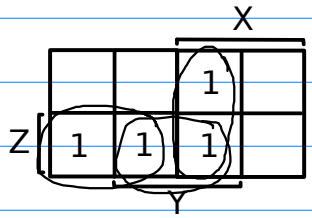
---

Absorption



$X + XY = X$

Consensus



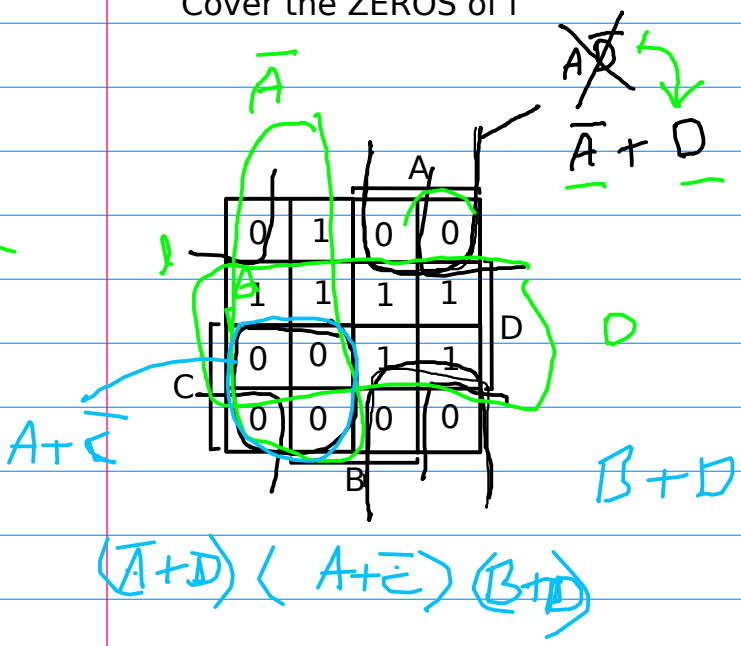
$f = XY + YZ + \bar{X}Z = XY + XZ$

$XYZ + X\bar{Y}Z$

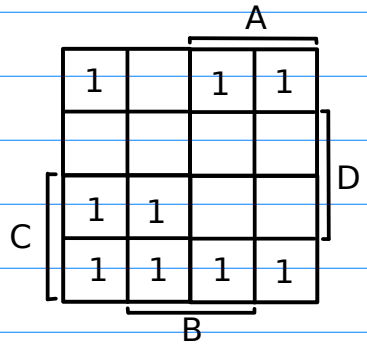
Recommendation: revisit previoius homework problems and visualize each operation in a K-map (draw)

Product of Sums minimization:

Cover the ZEROS of f



Compare to
Covering the ONES of f-bar



To get the minimum SOP/POS realization of a function, try both and pick the "best"